Mathematical Description: lecture 6

- The wave function describes the position of any particle in the medium at any time.
- For a sinusoidal wave, in which each particle undergoes simple harmonic motion about its equilibrium position,
  \[ y(x, t) = A \sin \omega(t - \frac{x}{v}) = A \sin 2\pi f (t - \frac{x}{v}) \]

  The disturbance travels a distance \( +x \) in an amount of time given by \( x/v \)
  \[ y(x, t) = A \sin(\omega t - kx) \]

Defining
\[ k = \frac{2\pi}{\lambda} \]

The Wave Equation

\[ \frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2} \]

with
\[ v^2 = \frac{\omega^2}{k^2} \]
Wave Speed on a String

\[ F = ma \]

\[ T \left[ \left( \frac{\partial y}{\partial x} \right)_{x+\Delta x} - \left( \frac{\partial y}{\partial x} \right)_{x} \right] = \mu \Delta x \frac{\partial^2 y}{\partial t^2} \]

Dividing through by \( \Delta x \) and taking the limit as \( \Delta x \to 0 \)

\[ \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} \]

so

\[ v = \sqrt{\frac{T}{\mu}} \]

Energy in Waves

\[ P(x,t) = F_x(x,t)v_y(x,t) = -F \frac{\partial y(x,t)}{\partial x} \frac{\partial y(x,t)}{\partial t} \]

\[ P(x,t) = \sqrt{\mu F} \omega^2 A^2 \cos^2 (\omega t - kx) \]

\[ P_{\text{max}} = \sqrt{\mu F} \omega^2 A^2 \]

\[ P_{\text{av}} = \frac{1}{2} \sqrt{\mu F} \omega^2 A^2 \]