Evolution of electrical current systems in the solar atmosphere: what are the key questions, and why?

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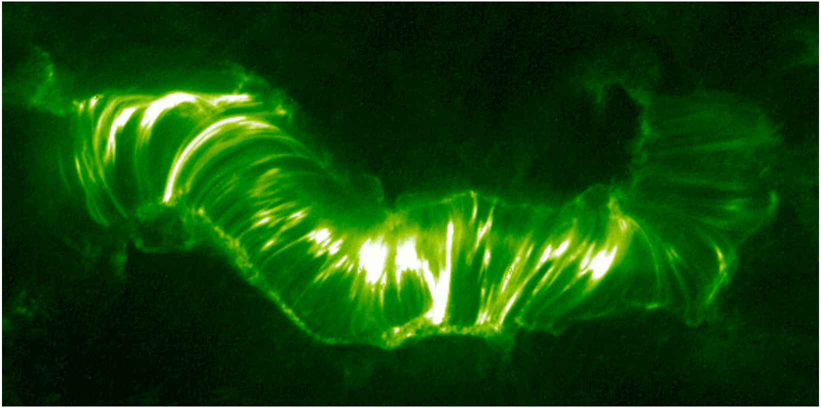
Background

Energy release in the corona

- Solar flares and CMEs involve release of magnetic energy
  - ‘release of free energy associated with currents’

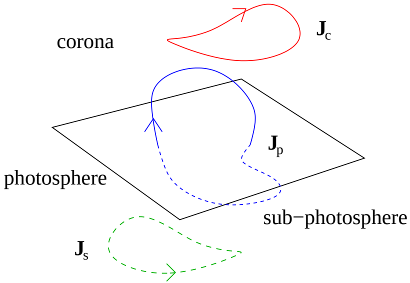


http://trace.lmsal.com/POD/

- Total energy

$$E = \frac{\mu_0}{8\pi} \int d^3x \int d^3x' \frac{\mathbf{J}(\mathbf{x}) \cdot \mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \tag{1}$$

- Current systems: $\mathbf{J}_\mathrm{c}$, $\mathbf{J}_\mathrm{p}$, $\mathbf{J}_\mathrm{s}$:



  - we don't know what changes during flares
  - consequently the 'free energy' is not well defined

- Free energy is often defined as $E - E_0$ where E_0 refers to the potential field $\mathbf{B}_0$ matching the observed B_n (e.g. Low 1982)
  - Thomson's principle: $\mathbf{B}_0$ is the minimum energy field for a fixed B_n
  - belief that photospheric B_n does not change during a flare
- Extremal energy state is a linear force-free field for fixed B_n and helicity (Woltjer 1958; 1959)
  - extremal energy state is a nonlinear force-free field when B_n is fixed and connectivity is specified (Sakurai 1989)
- However, flares may be driven by emerging flux, in which case B_n is changing? (e.g. Heyvaerts, Priest & Rust 1977; Zirin 1983)
- Can still estimate E, ΔE

- Poynting theorem (Maxwell's equations):

$$\frac{\partial}{\partial t}\left(\frac{B^2}{2\mu_0}+\frac{\epsilon_0 E^2}{2}\right)+\nabla\cdot\left(\frac{\mathbf{E}\times\mathbf{B}}{\mu_0}\right)=-\mathbf{J}\cdot\mathbf{E} \qquad (2)$$

  - $\mathbf{J}\cdot\mathbf{E}$ is the rate work is done per unit volume by the field
- Magnetohydrodynamics (simple Ohm's law):

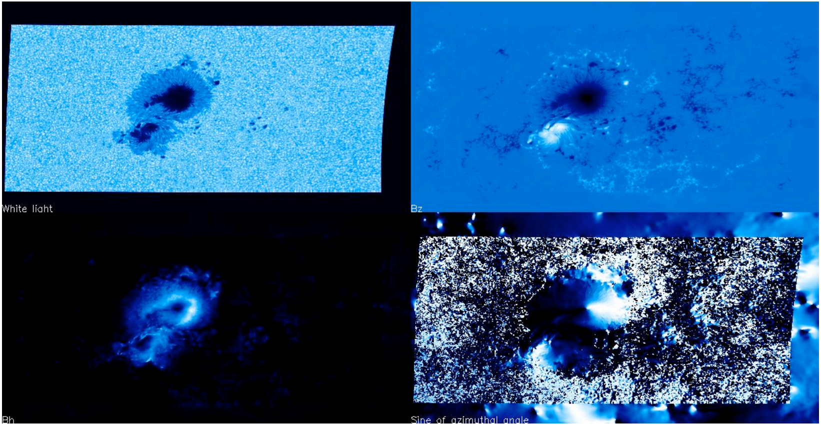
$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2+\frac{p}{\gamma-1}\right) + \nabla\cdot\left(\frac{1}{2}\rho u^2\mathbf{u}+\frac{\gamma}{\gamma-1}p\mathbf{u}\right)-\mathbf{J}\cdot\mathbf{E} = -\frac{J^2}{\sigma} \qquad (3)$$

  - $-J^2/\sigma$ describes dissipation
  - in MHD σ hides all sins

Observations

Photospheric fields

- Spectro-polarimetric measurements of photospheric lines permit inference of the vector magnetic field
  - it's an inference... caveat emptor
- New instruments (Hinode/SOT, SDO/HMI, SOLIS)



- Inversion problem: Stokes profiles $\rightarrow \mathbf{B}_{\mathrm{p}}$
  - least squares fitting to an analytic radiative transfer solution (e.g. Unno 1956; Rachovsky 1962; Auer, Heasley & House 1977; Skumanich & Lites 1987; Jefferies, Lites & Skumanich 1989)
  - model assumes Milne-Eddington atmosphere, constant physical quantities
  - observed profiles often do not have symmetries of the model
  - weak-field approximations used also (e.g. Jefferies & Mickey 1991)
  - More recent method: Stokes Inversion based on Response functions (SIR) (Ruiz Cobo & del Toro Iniesta 1992; Westendorp Plaza et al. 1998; 2001)
- 180 degree ambiguity in direction of $\mathbf{B}_{\mathrm{p}\perp}$
  - various methods of resolution (e.g. Metcalf et al. 2006)
- Error in spectro-polarimetric measurements

Photospheric currents

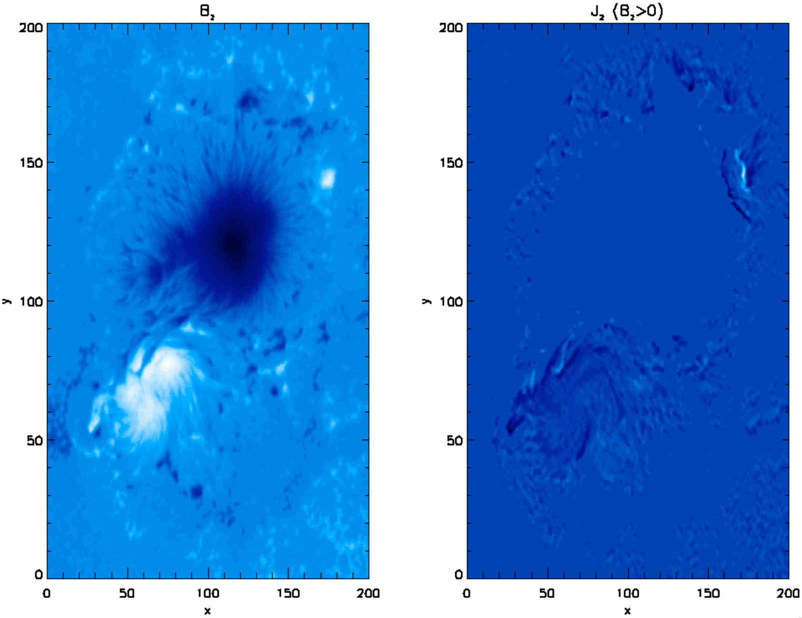
- Vertical component of current density (Moreton & Severny 1968)

$$J_{\mathrm{p}z} = \frac{1}{\mu_0}\left(\frac{\partial B_{\mathrm{p}y}}{\partial x} - \frac{\partial B_{\mathrm{p}x}}{\partial y}\right) \tag{4}$$

  - finite differencing of vector magnetic field values
  - substantial random errors (e.g. Leka & Skumanich 1999)
  - harder to categorise systematic uncertainty
  - some authors very skeptical (e.g. Parker 1996; cf. McClymont, Jiao & Mikic 1997; Semel & Skumanich 1998)
  - however, essentially the only quantitative information on current systems (vertical component of $\mathbf{J}_{\mathrm{p}}$)
- Components $J_{\mathrm{p}x}$, $J_{\mathrm{p}y}$ not available ($\partial \mathbf{B}/\partial z$ required)

Properties of currents

- Maximum values $J_{\mathrm{p}z} \approx 10 - 50\,\mathrm{mA\,m^{-2}}$ (Gary & Démoulin 1995; Leka & Skumanich 1999)
- Persist for $\approx$ 1 day (Pevtsov, Canfield, Metcalf 1994; Schrijver et al. 2005)
- Photospheric $J_{\mathrm{p}z}$ qualitatively consistent with coronal twist (Pevtsov, Canfield & McClymont 1997)
- Origin of photospheric currents is unknown
  - subphotospheric: flux emerges with currents flowing ('pre-stressed') (e.g. McClymont & Fisher 1989; Leka et al. 1996)
  - photospheric: coronal fields twisted by photospheric flows
- Photospheric model implies 'neutralized' patterns of current flow in a given polarity (e.g. Melrose 1991; Aulanier, Démoulin & Grappin 2005)
  - observations ambiguous (e.g. Wilkinson et al. 1992; Wheatland 2000)
  - Hinode data suggests currents are neutralized



B_z
J_z (B_z>0)
y
x
0
50
100
150
200

- Current helicity $H_{\rm c} = \int d^3x\, \mathbf{B} \cdot \mathbf{J}$
  - in common with magnetic helicity ($H_{\rm m} = \int d^3x\, \mathbf{A} \cdot \mathbf{B}$) a measure of twist
  - component of the integrand ($B_{{\rm p}z} J_{{\rm p}z}$) determined at photosphere
  - related to force-free parameter $\alpha_{\rm p} = \mu_0 J_{{\rm p}z} / B_{{\rm p}z}$
- Hemispheric preference for sign of current helicity (e.g. Richardson 1941; Seehafer 1990; Rust 1994)
  - left handed features in north, right-handed in south
  - however, localised variation in e.g. sign of $\alpha_{\rm p}$
- Mechanism responsible remains obscure (e.g. van Driel-Gesztelyi, Démoulin, & Mandrini 2003)

Modelling

Force-free fields

- Zeroth order static model

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \qquad \nabla \cdot \mathbf{B} = 0 \tag{5}$$

or

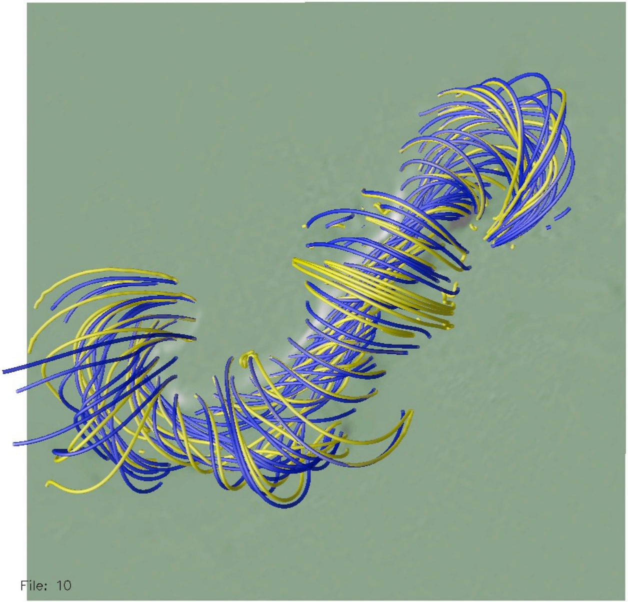
$$\nabla \times \mathbf{B} = \alpha \mathbf{B} \qquad \mathbf{B} \cdot \nabla \alpha = 0 \tag{6}$$

  - spatially varying α (nonlinear model) necessary
- Dependent variable **B** or else **B**, α
  - BCs: B_n and α on one sign of B_n
  - $\alpha_{\rm p} = \mu_0 J_{{\rm p}z}/B_{{\rm p}z}$
- Elliptic/hyperbolic PDEs may be numerically solved
  - current-field iteration (Grad & Rubin 1958)
  - magnetofrictional method (Chodura & Schlueter 1981)
  - vertical integration (Wu et al. 1990)
  - optimization method (Wheatland, Sturrock, Roumeliotis 2000)
  - fast current-field iteration (Wheatland 2006; 2007)

- Application to $\mathbf{B}_\mathrm{p}$ difficult
  - photospheric field is not force free (e.g. Metcalf et al. 1995) – preprocessing? (Wiegelmann, Inhester & Sakurai 2006)
  - difficulty of determining α_p
- Goals: estimate coronal energy, topology
  - MHD virial theorem provides energy from $\mathbf{B}_\mathrm{p}$:

$$E = \frac{1}{\mu_0} \int_\mathrm{p} (B_{\mathrm{p}x} x + B_{\mathrm{p}y} y) B_{\mathrm{p}z} dxdy \qquad (7)$$

    (Chandrasekhar 1961; Moldensky 1974)
  - assumes force-free BCs
- NLFFF workshops
  - 2005: Low & Lou test cases (Schrijver et al. 2006)
  - 2006: van Ballegooijien field (Metcalf et al. 2007)
  - 2007: Hinode/SOT AR 10930 (Metcalf/DeRosa, LWS 2007)



Metcalf et al. 2007

- Other limitations/problems:
  - difficulty of solution of PDEs
  - topologically significant sites not force-free?
  - sequence of force-free states is not equivalent to MHD evolution (e.g. Syrovatskii 1978)

Magnetohydrostatics (MHS)

- MHS is next simplest model

$$\nabla p = \mathbf{J} \times \mathbf{B} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \nabla \cdot \mathbf{B} = 0 \qquad (8)$$

  - p is constant on lines of $\mathbf{B}$ and $\mathbf{J}$
- Writing $\mathbf{J} = \mathbf{J}_\parallel + \mathbf{J}_\perp$ with $\mathbf{J}_\parallel = \alpha \mathbf{B}/\mu_0$

$$\mathbf{J}_\perp = \frac{1}{B^2}\mathbf{B} \times \nabla p \quad \text{and} \quad \mathbf{B} \cdot \nabla \alpha + \mu_0 \nabla \cdot \mathbf{J}_\perp = 0 \qquad (9)$$

  - BCs: B_n plus p, J_n on one sign of B_n (Grad & Rubin 1958)
- May be solved by Grad-Rubin iteration
  - start with $\mathbf{B}_0$
  - propagate boundary values of p along field lines
  - determine $\mathbf{J}_\perp$ everywhere
  - Solve for α along field lines given J_n and $\mathbf{J}_\perp$
  - Obtain a new $\mathbf{B}$ by solving $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ with $\nabla \cdot \mathbf{B} = 0$
  - iterate...

- Can be generalised to include gravity
  - ρ enters the force balance
  - p varies along field lines
  - equations (8) supplemented by EOS, assumption about T
- In principle could be applied to photospheric data
  - B_n, J_n from $\mathbf{B}_{\rm p}$
  - photospheric pressure diagnostic? (p_e from SIR?)
  - accommodates non force-free BCs
- Grad-Rubin iteration is a generalisation of current-field iteration (e.g. Wheatland 2006)
  - not significantly more computationally intensive

Magnetohydrodynamics (MHD)

- Minimum model for 'evolution of electrical current systems'
  - MHD modelling to date highly idealised
  - rarely applied to data (cf. Riley, Linker, Mikic 2001; Peter, Gudikson & Nordlund 2004; W. Abbett LWS 2007)
- Dependent variables **v**, **B**, p, ρ
  - $\mathbf{J} = \mu_0^{-1}\nabla \times \mathbf{B}$ considered a secondary quantity (e.g. Parker 1996)
  - simplistic BCs on **v**, **B** may imply unrealistic currents at boundaries, e.g. 'line tying' (Melrose 1991; 1995)
- Time dependent BCs
  - $\mathbf{B}_{\rm p}(t)$ available; $\mathbf{v}_{\rm p}(t)$, $p_{\rm p}(t)$, $\rho_{\rm p}(t)$ less accessible
  - correlation tracking for $\mathbf{v}_{\rm p}(t)$? (e.g. November & Simon 1988)
  - photospheric density/pressure diagnostics? ($p_e(t)$ from SIR?)
  - is it possible to perform real-time MHD modelling from data?
- Numerical solution of equations remains challenging
  - hyperbolic/parabolic PDEs

Conclusions

Summary

- Coronal currents determine magnetic energy and dissipation
- Observing current systems is problematic
  - $J_{\mathrm{p}z}$ is inferred with substantial uncertainty
  - nature of currents not well understood
  - new instruments should provide substantially improved data
- Limited ability to model coronal fields, currents from BCs
  - force-free, MHS and MHD models described
  - prospects for application to data discussed

Questions

- Can we better define 'free energy'?
- Can we improve Stokes inversion?
  - can we extract additional information, e.g. p_e?
- Can we confirm the reality of inferred values of $J_{\mathrm{p}z}$?
- Can we identify the origin of the currents?
- Can we perform coronal field modelling from BCs?
  - is force-free modelling useful/possible?
  - would magnetostatic modelling be better?
  - is time-dependent MHD modelling possible?