DO SOLAR FLARES EXHIBIT AN INTERVAL–SIZE RELATIONSHIP?

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Abstract. Some models for flare statistics predict or assume that there is a relationship between the times between flares and the energy of flares. This question is examined observationally using the WATCH solar X-ray burst catalogue. A rank correlation test applied to the data finds strong evidence for a correlation between the time since the last event, t_b, and the size (peak count rate) of an event, and for a correlation between the time to the next event, t_a, and the size of an event. A more sophisticated statistical test, taking into account a probable bias in event selection, does not support the hypothesis that event size depends on t_b or t_a.

1. Introduction

Solar flares are explosive events in which magnetic energy is liberated in the solar corona. Statistical studies of flares based on soft and hard X-ray observations provide valuable clues to the puzzle of how a flare occurs. Particular attention has been focused on the observational finding that the frequency-energy distribution of flares is a power law (e.g., Hudson, 1991), and a number of models have been proposed to explain this result, including the avalanche model (Lu and Hamilton, 1991), and the older Rosner-Vaiana (RV) model (Rosner and Vaiana, 1978) and variants thereof (Litvinenko, 1994, 1996; Aschwanden, Dennis and Benz, 1998). Recently a general mathematical formalism describing the avalanche model and the RV-type models was presented by Wheatland and Glukhov (1998). The statistical models are designed to reproduce the power-law frequency-energy distributions of flares, but they may be tested by examining other statistical properties of flares. For example, the waiting-time distribution, or the distribution of times between events, provides information about whether events are independent of one another (as assumed by RV-type models, and predicted by the avalanche model). Based on International Cometary Explorer/International Sun-Earth Explorer 3 (ICE/ISEE-3) observations, Wheatland, Sturrock, and McTiernan (1998) found that hard X-ray bursts near in time show an interdependence (in the sense that the occurrence of one burst makes another burst relatively more likely), although this may be reconciled with the models if the dependent bursts belong to the same flare.

This paper examines the question of whether there is a relationship between the times between flares and the energy of flares. If such a relationship exists, it provides insight into energy storage in the solar corona. For example, the astrophysical

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sources known as X-ray bursts show a correlation between the time interval since the previous burst and the integrated burst energy (Lewin, Van Paradijs, and Taam, 1995). The interpretation is that the burst is due to a thermonuclear explosion of material accreted from one compact object to another, and for longer time intervals between bursts, more material has accreted and is available to burn. In the case of solar flares, it is not known how energy is transferred to and stored in the solar corona, and statistical studies may provide clues to the underlying processes.

Another motivation for this study is that models for flare statistics make specific predictions or assumptions about the interval-energy relationship, and so an examination of the relationship can in principle distinguish between the models. Specifically, the avalanche model predicts no relationship (Lu *et al.*, 1993), whereas the RV-type models predict a relationship similar to the X-ray burst relationship, namely that the energy of a flare should depend on the time since the previous burst. If there is an interval-energy relationship, it plays a part in determining the flare frequency-energy distribution, as follows. Suppose the energy, E of an event depends on the time t_b since the previous event, or alternatively, depends on the time t_a to the next event. (The subscripts b and a denote 'before' and 'after,' respectively.) The rules for combining probabilities give the probability distribution function for observing an event of energy E, $\mathrm{prob}(E)$:

$$\mathrm{prob}\ (E) = \int_0^\infty \mathrm{prob}\ (E|t_i)\ \mathrm{prob}\ (t_i)\ \mathrm{d}t_i\ , \tag{1}$$

where $i = a$ or $i = b$, $\mathrm{prob}(E|t_i)$ denotes the probability of an event with energy E occurring (per unit energy) given an interval t_i, and $\mathrm{prob}(t_i)$ is the waiting-time distribution. The flare frequency-energy distribution is given by

$$\mathcal{N}(E) = \lambda\ \mathrm{prob}\ (E)\ , \tag{2}$$

where $\lambda = [\int_0^\infty t_i\ \mathrm{prob}\ (t_i)\ \mathrm{d}t_i]^{-1}$ is the mean rate of flaring. Hence the frequency-energy distribution is determined by the interval-energy relationship $\mathrm{prob}(E|t_i)$ together with the waiting-time distribution, provided that an interval-energy relationship exists. The RV model illustrates these ideas. In that model, the available flare energy increases exponentially between flares, and each flare releases all of the stored energy. Hence

$$\mathrm{prob}\ (E|t_b) = \delta\left[E - E_0(e^{\alpha t_b} - 1)\right]\ , \tag{3}$$

where E_0 is the ground-state energy of the system, and α is the rate of increase of energy, per unit energy. Flaring is assumed to be a Poisson process in time, and so the waiting time distribution is exponential:

$$\mathrm{prob}\ (t_b) = \lambda e^{-\lambda t_b}\ . \tag{4}$$

Substituting Equations (3) and (4) into (1) and evaluating the integral (by a change of variable) gives the frequency-energy distribution in the RV model,

$$\mathcal{N}(E) = \frac{\lambda^2}{\alpha E_0} (1 + E/E_0)^{-(1+\lambda/\alpha)} \ . \tag{5}$$

The topic of this paper has been looked at before by a number of authors. Biesecker (1994) examined the time history of flares observed with the Burst and Transient Source Experiment (BATSE) on the Compton Gamma Ray Observatory (GRO), and found no evidence for a relationship between the peak rate of the flare and the time to the previous flare, or the time to the next flare. One shortcoming of this study was that flares were not identified with individual active regions, and including times between flares from different active regions might mask an interval – size relationship, if it were present. Hudson *et al.* (1998) observed a small number of flares from a single active region using the Soft X-ray Telescope (SXT) on the Yohkoh spacecraft, but did not report a significant correlation in between the peak flux and the time to preceding or following flares. Finally, Crosby *et al.* (1998a; see also Crosby, 1996) used the Danish Wide Angle Telescope for Cosmic Hard X-rays (WATCH) on the Russian GRANAT satellite and identified deka-keV solar X-ray bursts with their originating active regions by comparison with GOES data. They reported no evidence for a relationship between the time between bursts and the size of bursts, although they did not appear to apply quantitative tests.

In this paper the WATCH/GRANAT data, now available as the WATCH solar X-ray burst catalogue (Crosby *et al.*, 1998b), is revisited (Section 2.1). In Section 2.2 simple correlation tests are applied to the data to determine if there is a relationship between burst intervals and the energy of the bursts. In Section 2.3 a more sophisticated test is used, that takes into account a suspected bias in the data. Finally, in Section 3 the results of the tests are discussed.

2. Data and Analysis

2.1. Data

The details of the WATCH solar X-ray burst catalogue are described in Crosby *et al.* (1998b). The data include start, end and peak times, total counts and the accumulation time for the counts, background estimates, and active region identification (where possible) for 1551 X-ray bursts observed over the years 1990 – 1992. In this study the events were first grouped according to active region number. For each event in a given active region it was determined whether there was a preceding event without interruption from data gaps. If so, a data point $(\mathcal{F}, t_b)$ was obtained, where $\mathcal{F}$ is the peak count rate (flux) of the event, and t_b is the time to the preceding event. A similar procedure was followed to give points $(\mathcal{F}, t_a)$, where t_a is the time to a following event.

A plot of t_b versus $\mathcal{F}$ for data pairs obtained for all observed active regions is shown in Figure 1. (This plot is similar to Figure 13 in Crosby *et al.* (1998a).) Similarly Figure 2 shows a plot of t_a versus $\mathcal{F}$ for data pairs for all active regions.

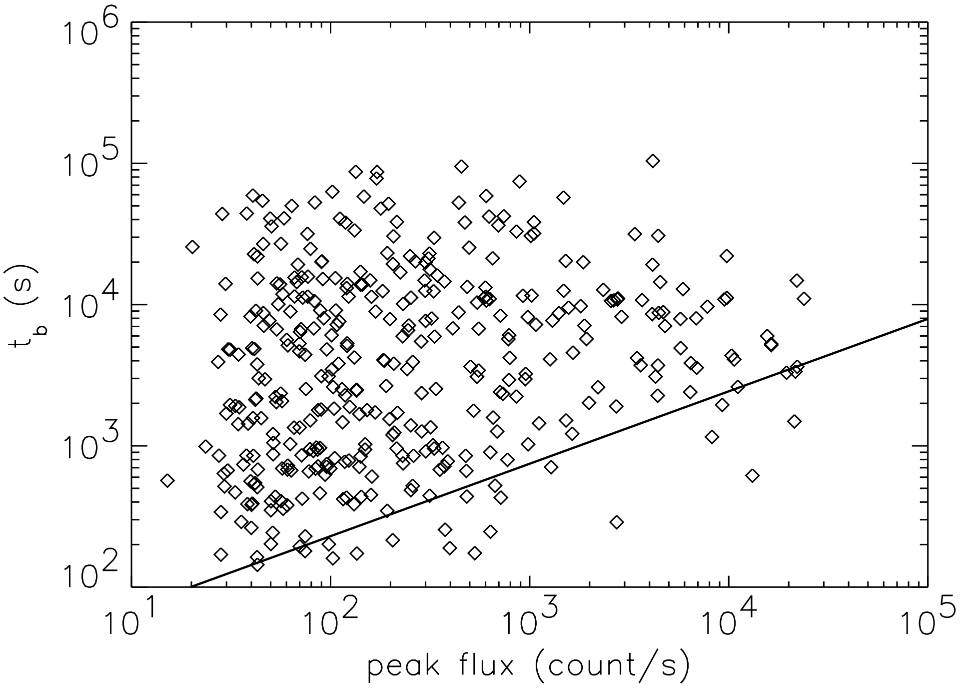

Figure 1. Plot of time to previous event, t_b versus peak count rate for events in a given active region.

The significance of the lines in the lower right of each figure will be explained in Section 2.3.

2.2. Rank correlation

Figures 1 and 2 do not show obvious correlations between the time intervals between events and the peak count rates of events. However, both figures appear to have relatively few large events with corresponding small intervals (i.e., an absence of points in the lower right corner).

A standard test for the independence of data with unknown distributions is provided by the Spearman rank-order correlation coefficient, r_s (Press *et al.*, 1992). For Figure 1, $r_s = 0.22$. The probability of getting a value of $|r_s|$ larger than this from the same number of independent datapoints is about 1.0×10^{-5}. Hence a simple statistical test indicates that the datapoints $(\mathcal{F}, t_b)$ are extremely unlikely to be independent! Similarly for Figure 2, $r_s = 0.10$. The probability of obtaining a value of $|r_s|$ larger than this from the same number of independent datapoints is about 8.9×10^{-3}, i.e., just less than one percent. Hence the rank correlation test suggests that $(\mathcal{F}, t_a)$ are also not independent, although the result is not as significant as that found for $(\mathcal{F}, t_b)$.

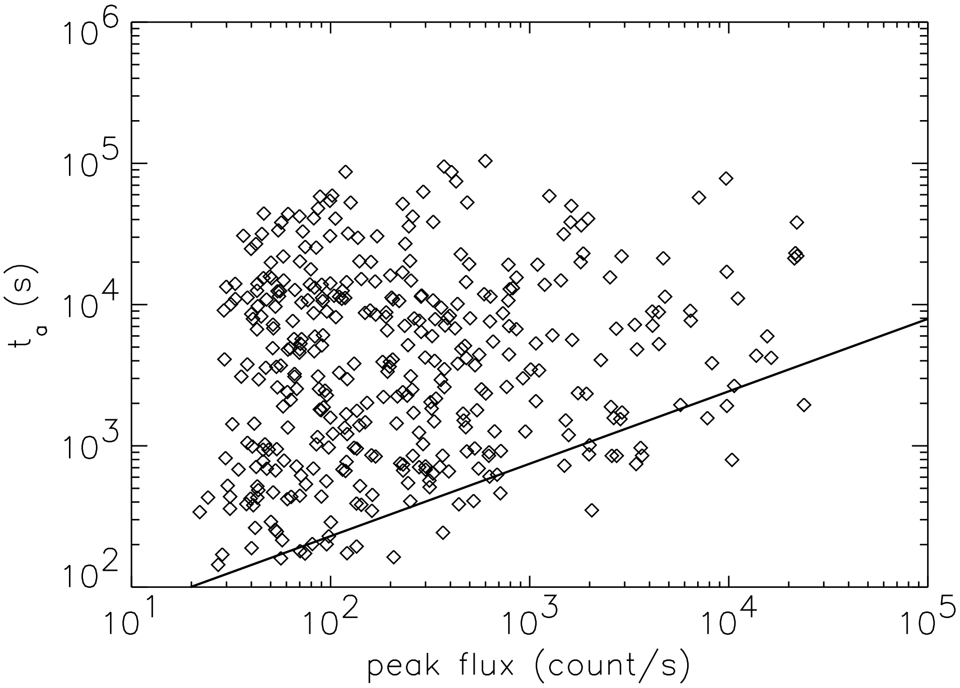

Figure 2. Plot of time to next event, t_a versus peak count rate for events in a given active region.

2.3. Consideration of Bias

At face value these results provide evidence that the size of an event is related to the time to the preceding event, and also perhaps to the time to the following event. However, it is important to consider whether biases are influencing the result of the correlation tests.

Figure 3 illustrates the relationship between the peak count rate of the WATCH bursts, $\mathcal{F}$ and the duration of the bursts, T. There is a very clear correlation between these quantities, so that bigger bursts tend to last longer. The solid curve shows the best fit power law to all of the points: $T = 21.6\mathcal{F}^{0.51}$, in the chosen units. This relationship will lead to an important bias in the interval – size diagrams: namely, if two (distinct) large bursts occur close together in time, their large durations may mean that they overlap, and they may then be counted, in the selection procedure, as a single burst. This will result in a relative absence of short interval/large count rate points in both the $(\mathcal{F}, t_b)$ and $(\mathcal{F}, t_a)$ diagrams, as is observed (Section 2.2).

To decide whether this effect could produce the observed interval–size dependency, it is necessary to model the bias and then account for it in the test for independence. The bias may be modelled, in a crude way, by replacing the peak count rate/duration scatter diagram by the line of best fit (see Figure 3), and then assuming that bursts closer in time than the duration implied by the line of best fit are not observed in the interval-size diagrams. The line of best fit to the $\mathcal{F}$ versus

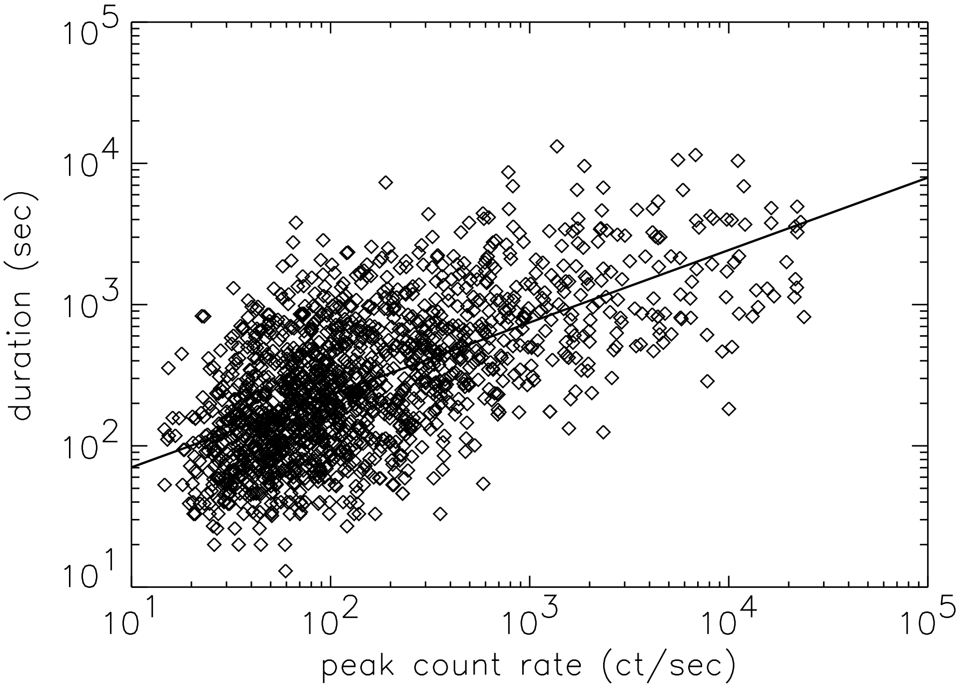

Figure 3. Plot of duration of event versus peak count rate for all events. The solid curve is the power law of best fit.

T diagram has been drawn on Figures 1 and 2: it is the solid curve in the lower right. The model of the bias is the assumption that points to the right and below this line are not observed, i.e., in Figure 1 points with $t_a < 21.6\mathcal{F}^{0.51}$ are missed, and in Figure 2 points with $t_b < 21.6\mathcal{F}^{0.51}$ are missed. These truncations of the datasets are not parallel to the axes, and so will affect the result of correlation tests applied to the data.

To account for the bias, we use the test of independence for truncated data proposed by Efron and Petrosian (1992; see also Lee, Petrosian, and McTiernan, 1993). The test involves calculating a statistic t_1, that is the sum of normalized differences between the actual rank and the expected rank of each point, where the ranks are calculated in a way that takes into account the truncation of the data. For independent datapoints, the statistic t_1 is normally distributed, with mean zero and unit standard deviation.

The test was applied to the data in Figures 1 and 2. For Figure 1 the value of the statistic is 1.75. The chance of getting a larger value of $|t_1|$ from independent datapoints is $\mathrm{erfc}(|t_1|/\sqrt{2})$, which evaluates to about 8%. Once the suspected bias is taken into account, there is no strong evidence for a dependence between the size of a burst and the time to the preceding burst. This shows that the correlation detected in Section 2.2 is entirely due to the relative absence of datapoints in the lower right of the figure, as suspected. Similarly, applying the Efron–Petrosian test to Figure 2 we obtain the value of the statistic $t_1 = 1.21$. The probability of getting a larger value of $|t_1|$ from independent data is 23%, so once again there is no evidence for a dependency.

3. Discussion

In this paper the question of whether solar flares exhibit an interval–size relationship has been addressed by examining X-ray events in the WATCH solar X-ray burst catalogue. The chief advantage of the dataset is that bursts are identified with individual active regions, so that time intervals between flares in a given active region can be examined. Rank correlation tests were found to give a very significant correlation between the peak count rate of a burst, $\mathcal{F}$ and the time to the previous burst, t_b, and a significant correlation between $\mathcal{F}$ and the time to the next burst, t_a. A bias was identified in the data, namely that because larger bursts have longer durations, if two large bursts occur close together they are likely to overlap, and may then be counted as a single burst. This bias was modelled as a truncation of the dataset for points with intervals (t_b or t_a) less than the duration, T of a burst for a given count rate. The best fit to the T vs. $\mathcal{F}$ diagram was used to specify a definite relationship between duration and count rate. Assuming this known truncation of the data, a statistical test for independence in the presence of truncation was applied. This test found no evidence for a relationship between $\mathcal{F}$ and t_b, or between $\mathcal{F}$ and t_a. Since the bias is believed to be present in the data and to be reasonably well-modelled by the procedure adopted in Section 2.3, the conclusion is that the data does not support an interval–size relationship. This result is consistent with the other studies mentioned in the Introduction.

There are two possible interpretations of the result. Either there is an interval-size relationship and it is being masked by various effects, or there is no interval-size relationship in flares. Consider first the possibility of masking. If an active region contains a number of independent flaring elements, each of which produces events with a definite interval–size relationship, then observations of flares from the entire active region will include intervals between flares from the distinct flaring elements, and so the interval–size relationship will be disguised, as suggested by Rosner and Vaiana (1978) and Lu (1995). This possibility could be tested by looking for an interval-size relationship in a sequence of homologous flares, i.e., flares that recur in the same location within an active region. A second effect that could mask an interval–size relationship is the missing of small events due to instrumental sensitivity, a threshold for detection, or because of background emission. In this case, the intervals t_b and t_a would be observed to be larger than their true values. If there is a definite interval–size relationship, i.e., a curve in the t_b vs. $\mathcal{F}$ or t_a vs. $\mathcal{F}$ plots, the observed points would lie above the curve, which could make the detection of the underlying relationship difficult. This possibility can be tested by using observations with greater sensitivity, e.g., observations of microflaring. Active region transient brightenings have energies comparable to the smallest WATCH bursts and occur repeatedly in individual soft X-ray loops (Shimizu *et al.*, 1992). The time history of transient brightenings does not appear to have been tested for an interval–size relationship. A final possibility as regards masking of an interval–size relationship is that the relationship is not definite, but probabilistic. In that case, it may be intrinsically hard to detect, especially given the other effects, in particular a failure to detect small events.

The simplest interpretation of the results presented above, and the one favored here, is that there is no interval–size relationship for flares. The avalanche model does not exhibit an interval–size relationship (Lu, 1995), so the results of this paper are consistent with the avalanche model. RV-type models are excluded in this interpretation. Other flare models may be consistent with the results above – in particular, models which do not involve in situ storage of energy in the corona. An example is the ‘colliding wave-packet’ model of Uchida and Shibata (Uchida and Shibata, 1988), which has recently been re-examined as a model to account for the statistics of both flares and of active region transient brightenings (Wheatland and Uchida, 1999).

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