Coffee cooling

A mug of coffee cools from $100\ °\mathrm{C}$ to room temperature, $20\ °\mathrm{C}$. The mass of the coffee is $m = 0.25$ kg and its specific heat capacity may be assumed to be equal to that of water, $c = 4190\ \mathrm{J.kg^{-1}.K^{-1}}$.
Calculate the change in entropy
(i) of the coffee
(ii) of the surroundings

Solution

The entropy of both the coffee and surroundings will change, so we have to calculate them separately. For each, we need to find the initial and final states A and B, and calculate the entropy change for a reversible process that takes us between the same states.

Coffee: cools from $T_1 = 100\ °\mathrm{C} = 373$ K to $T_2 = 20\ °\mathrm{C} = 293$ K. Consider heat being transferred to the surroundings at an infinitesimal rate

$$dQ = mc\, dT$$

Then the total entropy change will be

$$\Delta S = \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{mc\, dT}{T} = mc \int_1^2 \frac{dT}{T}$$

Hence

$$\Delta S_{\text{coffee}} = mc\ \ln\frac{T_2}{T_1} = (0.25)(4190)\ln\frac{293}{373} = -253\ \mathrm{J.K^{-1}}$$

i.e. entropy of the coffee *decreases*, as expected, since heat is flowing out.

Surroundings: Heat flow Q goes into the surroundings, while they remain at 293 K (large heat sink). We calculate Q by working out how much heat the coffee loses:

$$Q = mc\Delta T = (0.25)(4190)(373 - 293) = 8.38\times10^4\ \mathrm{J}$$

So the entropy change for the surroundings, at constant temperature $T = 293$ K, is

$$\Delta S_{\text{surroundings}} = \frac{8.38\times10^4}{293} = +286\ \mathrm{J.K^{-1}}$$

which is positive, and larger than the entropy decrease of the coffee. The total entropy change is

$$\Delta S_{\text{tot}} = \Delta S_{\text{coffee}} + \Delta S_{\text{surroundings}} = -253 + 286 = +33\ \mathrm{J.K^{-1}}$$

which is positive, as expected (irreversible process).