Standing, walking, running, and jumping on a force plate

Rod Cross
Physics Department, University of Sydney, Sydney, New South Wales 2006, Australia

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Details are given of an inexpensive force plate designed to measure ground reaction forces involved in human movement. Such measurements provide interesting demonstrations of relations between displacement, velocity, and acceleration, and illustrate aspects of mechanics that are not normally encountered in a conventional mechanics course, or that are more commonly associated with inanimate objects. When walking, the center of mass follows a curved path. The centripetal force is easily measured and it provides an upper limit to the speed at which a person can walk. When running, the legs behave like simple springs and the center of mass follows a path that is the same as that of a perfectly elastic bouncing ball.

I. INTRODUCTION

Force plates are commonly used in biomechanics laboratories to measure ground forces involved in the motion of human or animal subjects.[1–4] My main purpose in this article is to show that a force plate can be used as an excellent teaching aid in a physics class or laboratory, to demonstrate relationships between force, acceleration, velocity, and displacement. An additional purpose is to show that walking, running, and jumping provide interesting applications and illustrations of elementary mechanics, not normally encountered in a conventional mechanics course. When lecturing to a class of students on mechanics, teachers often use the nearest object, such as a piece of chalk or a blackboard eraser, to show that an object will accelerate when subject to a force, or that the resultant force is zero on an object at rest. With a force plate, a much more dramatic and entertaining series of demonstrations is possible, since the ''object'' can be the lecturer or a student. Furthermore, the force wave form can be displayed and measured directly to demonstrate either qualitative or quantitative relations between force, acceleration, and displacement. The combination of measuring, seeing, and feeling the forces involved must strengthen the connection between the experienced world and the world measured by instruments. This approach, and the topic itself, is particularly appropriate for life-science students, but would benefit all students.

A force plate is simply a metal plate with one or more sensors attached to give an electrical output proportional to the force on the plate. The sensor can either be a strain gauge or a piezoelectric element. As such, it performs the same function as a bathroom weighing scale, but such a scale is not suited for dynamic force measurements either because the readout is digital or because the pointer moves too quickly and the spring mechanism vibrates excessively in response to small changes in the force. At frequencies less than about 100 Hz, the output of a force plate is accurately proportional to the applied force and can be monitored on a storage oscilloscope or fed to a data acquisition system for display and further analysis. Force plates are generally not suited for studying impacts of duration less than a few ms, since most plates are large and flexible and vibrate at a frequency of about 400 Hz. These vibrations are normally of no consequence in biomechanics applications since they are strongly damped if foot contact is maintained for several ms or more.

A typical application of a force plate is to measure the ground reaction force on each foot while walking. The vertical component of the force rises from zero to a maximum value of about Mg, then drops below Mg, then rises again to about Mg, then drops to zero. This time history reveals some fascinating details of the walking process that are even now still not fully understood. The dip in the middle of the force wave form is due to the centripetal force associated with motion of the center of mass along a curved path. The second peak is smaller than the first when walking at a fast pace, but I was unable to find a satisfactory explanation of this effect in the literature. The reader is challenged, before reading below, to predict the magnitude and direction of the horizontal component of the force on each foot. There are literally hundreds if not thousands of books and papers related to the biomechanics of human and animal walking, and about the same number on running. Force plate measurements feature prominently in a large fraction of these papers.

II. THE FORCE PLATE

A fully instrumented force plate in a biomechanics laboratory usually costs at least \$30 000. A much simpler and cheaper ''homemade'' version can be constructed for teaching purposes. The plate used to present the results in this paper is shown in Fig. 1. It does not require a power supply, and is calibrated simply by standing on it. It consists of two parallel aluminum plates, each 16 mm thick, with four piezoelectric elements sandwiched between the plates, one in each corner. The four elements were connected in parallel by direct contact with the upper and lower plates. The assembly was bolted together with four nylon bolts to maintain firm contact with the piezos. Clearance holes in the top plate, around the nylon bolts, allowed the top plate to flex freely under load. The dimensions of each plate were 365 mm×250 mm, sufficient to locate two feet on the plate. Most force plates used in research laboratories are about twice this size so that a person can land on the plate without any deliberate change in stride. Force plates used for research also contain sensors that respond to forces in the horizontal plane. Such sensors were not incorporated in the homemade plate since the horizontal forces are usually much smaller than the vertical component for the movements studied, and since it would require a more complex arrangement of mounting the upper plate.

Piezo disks used in piezo buzzers were considered for the force plate application since they are cheap and generate a large output voltage, but were found to be unsuitable since

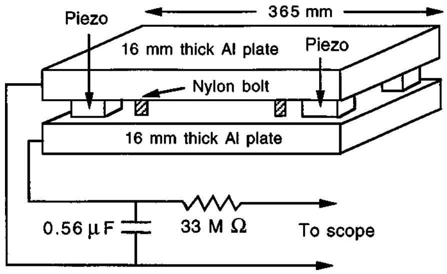

Fig. 1. The homemade force plate. A thin sheet of rubber was glued to the outer surfaces of both plates to prevent slipping.

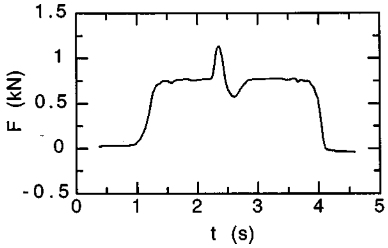

Fig. 2. The wave form observed when a person steps onto the force plate, in a crouching position, then stands up straight, then steps off the force plate.

the output of such a disk saturates with a load greater than about 0.5 kg. The piezos used in the force plate were manufactured for sonar applications, cost \$25 each, and were designed to operate without saturation in the high-pressure environment at the bottom of the ocean.[5] They were made from a ''hard'' PZT ceramic in the form of square blocks each 28 mm×28 mm×5 mm, with a silvered electrode covering each of the two large surfaces. The output voltage of a piezo element is given by $V=Q/C$, where C is the capacitance of the element, $Q=d_{33}F$ is the charge induced by a force F applied in a direction perpendicular to the electrode surfaces, and d_{33} is the relevant piezo coefficient (340×10^{-12} C/N for the material chosen). Each piezo had a capacitance $C=1.95\pm 0.01$ nF and generated an output voltage of 174 mV per Newton.

The capacitance of the assembly was 7.80 nF, but it was artificially increased to 0.57 μF by connecting an external 0.56 μF capacitor, as shown in Fig. 1. The output signal from the assembly was monitored using a 33 MΩ resistor in series with a 1 MΩ input impedance digital storage oscilloscope to increase the RC time constant to 19 s. The output signal was therefore reduced in amplitude by a factor of 73 by the external capacitor, and by an additional factor of 34 due to the series resistor, to give an overall sensitivity of 17.5 mV per kN, the force on each piezo being reduced by a factor of 4 since four piezos were used to share the load. To monitor the force wave form on a longer time scale and/or at higher sensitivity, force plates are normally connected to a charge amplifier. This was not necessary for the experiments described below, since a force was applied to the plate for only a few seconds. The signal generated by the force due to the nylon bolts decayed to zero with a time constant of 19 s. The output signal observed from the force plate therefore represents the change in the force, above that due to the bolts, provided the additional force is applied for a period significantly less than 19 s.

If a force is applied to the center of the plate, each piezo will share the load equally and generate the same charge. An off-center force results in an unequal sharing of the load. In commercial force plates the four piezo outputs are monitored separately and processed electronically to provide measurements of the total force as well as the line of action of the force. In the present case, all four piezos were connected directly in parallel. Each piezo generates a charge proportional to the local force on the piezo. When all four piezos are connected in parallel, the output signal is proportional to the total charge and is hence proportional to the total force on the plate. Consequently, the same signal is generated regardless of whether one stands in the middle of the plate or on one corner, provided all four piezos are closely matched in sensitivity.

III. STANDING AND JUMPING WAVE FORMS

If one stands with both feet on a force plate, it will register a force $F=Mg$. If the center of mass (CM) is then lowered by bending the knees, the force does not remain equal to Mg. Instead, the force decreases ($F<Mg$) and then increases ($F>Mg$) before settling back to Mg. Alternatively, if one steps onto the force platform in a crouching position and then stands up straight, the result shown in Fig. 2 is obtained. In raising the CM, the CM starts with zero speed, accelerates to finite speed, then decelerates to a new resting position. During this maneuver, the plate registers a force F given by $F-Mg=Ma$, where a is the acceleration of the CM vertically upward.

Figure 3 shows the wave form observed when jumping off the floor and dropping onto the force plate from a height of a few cm, landing with both feet simultaneously. The force rises rapidly to a value significantly larger than Mg. The magnitude of the impact force can be reduced by allowing the knees to bend more on contact, or increased by keeping the legs straight. The CM has a negative velocity at contact, decreasing rapidly to zero with a slight positive velocity overshoot due to flexure of the knees. The initial acceleration is therefore large and positive in a direction vertically upward, so $F>Mg$ initially. This is a good example where a deceleration in one direction can usefully be interpreted as an acceleration in the opposite direction. The magnitude of the force is easily calculated, using estimates of the initial velocity and the time taken to come to rest. Figure 4 shows the wave form that results when jumping off the platform from a standing start. The initial response corresponds to a slight

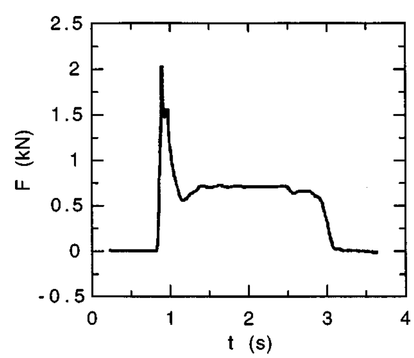

Fig. 3. The wave form observed when jumping off the floor onto the force plate and then stepping off the plate.

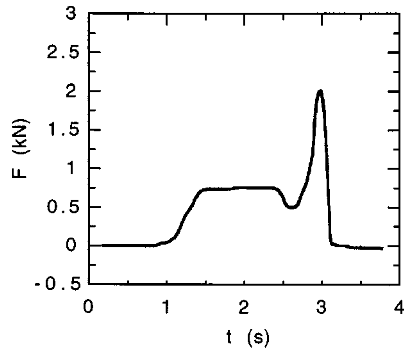

Fig. 4. The wave form observed when stepping onto and then jumping off the force plate.

lowering of the CM, in preparation for the jump. The impulse is simply related to the height of the jump, which was about 3 cm in this case.

IV. WALKING WAVE FORMS

Walking or running is not a topic that is usually studied in a physics course, but it is ideally suited for a class of life-science or sports science students. It illustrates some interesting aspects of elementary mechanics in a way that should appeal to all physics students. For example, when walking or running at constant speed, no horizontal force is required, at least in a time-average sense. Why then do we keep pushing backward on the ground to maintain speed, and why do we not keep accelerating with every stride? The answer is well known in the biomechanics field, but it is probably less well known by physicists. Wind resistance is not sufficient to supply the retarding force required to keep the speed constant. Top sprinters push backward on the ground with a force comparable to Mg. If wind resistance were the only retarding force, and if their legs could move fast enough, sprinters would reach a terminal velocity comparable to the freefall speed of a person jumping from a plane.

The main retarding force in both walking and running arises from the fact that the front foot pushes forwards on the ground, resulting in an impulse that is equal and opposite the impulse generated when the back foot pushes backwards. This is shown schematically in Fig. 5. The instantaneous walking speed therefore fluctuates, but if the average speed remains constant, then the average horizontal force remains zero. The retarding force can therefore be attributed to a frictional force, between the front foot and the ground, that prevents the front foot sliding forward along the ground.[6] This result is surprising since it means that top sprinters, as well as slow walkers, must spend about half their time push-

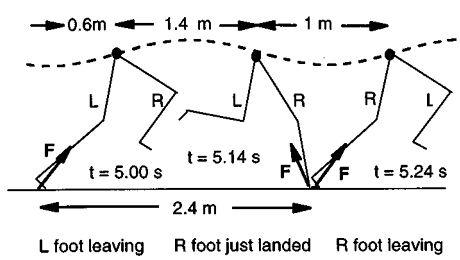

Fig. 5. A schematic diagram showing the ground forces when running, and the vertical displacement of the CM (dashed line). Also shown are typical distances and times when running at 10 ms^{-1}.

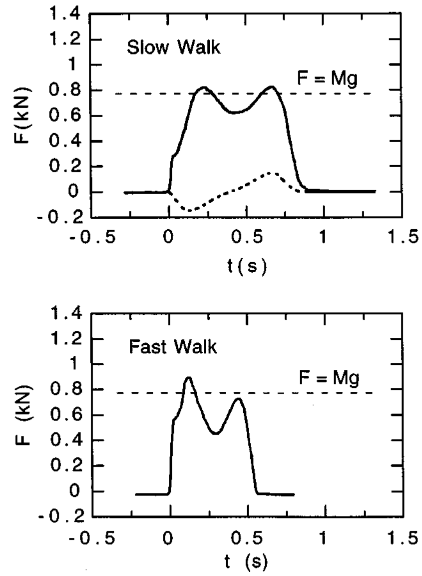

Fig. 6. The wave forms observed when walking (a) at a slow pace and (b) a fast pace. The dotted line represents the horizontal component of the ground force, in the direction of the gait.

ing forward on the ground in order to slow down. It is necessary to slow down, by bringing the front foot to rest, so that the back foot can catch up to and then pass in front of the CM. While the front foot is at rest, the back foot travels at about double the speed of the CM. The maximum speed of a runner is therefore limited to about half the speed at which the back foot can be relocated to the front.

The vertical component of the force acting on one foot when walking on the force plate is shown in Fig. 6. Also shown is the horizontal component in the direction of the gait. The latter component was not measured with the home-made force plate, but the wave form is well documented in the biomechanics literature.[1–4,7–13] The ratio of the vertical to the horizontal component indicates that the line of action of the ground force acts through a point that is close to the CM at all times. In this way, the torque about the CM remains small and the walker or runner can maintain a good balance.

The vertical force wave form is interesting since it has two distinct peaks where $F \approx Mg$ and a dip in the middle where $F < Mg$. Both peaks are similar in amplitude when walking at a slow or medium pace, but the second peak is smaller than the first when walking at a fast pace. The force rises from zero as weight is transferred from the back to the front foot, and returns to zero when weight is transferred back to the other foot at the end of the stride. The force wave form can be interpreted in terms of the vertical motion of the CM, provided one adds the force on both feet to obtain the resultant force, as shown in Fig. 7. It is reasonable to assume that the left and right wave forms are identical for most people. With the aid of two force plates, it is found that the timing of the ground forces on the left and right feet are such that the first force peak observed on each foot coincides with the time at which the back foot lifts off the ground, and the second force peak coincides with the time at which the other foot first lands on the ground. The vertical acceleration, a, is obtained by subtracting Mg from the force wave form. Integration of a yields the vertical velocity, v, and integration of v yields the vertical displacement, z. The constants of inte-

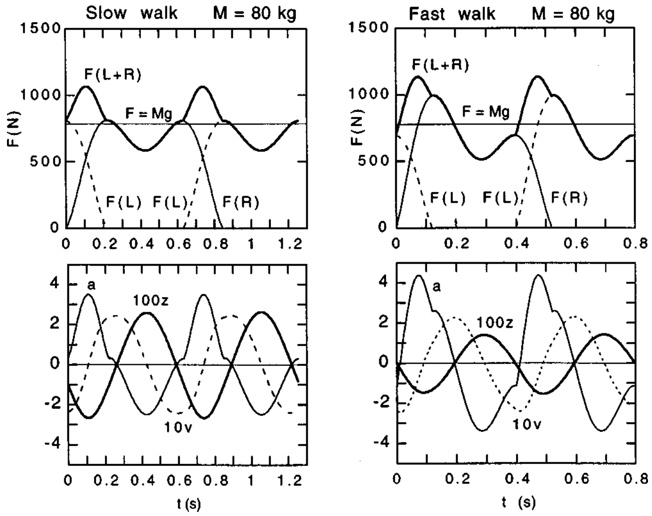

Fig. 7. The vertical component of the ground force $F(L+R)$, obtained by adding the force due to the left foot $F(L)$ and the force due to the right foot $F(R)$. These results were used to calculate the acceleration a (ms^{-2}), the velocity v (ms^{-1}), and the vertical displacement z (m).

gration can be chosen so that the time averages of a and v are zero, and the time average of z is either zero or its measured value.[7]

The results of such a calculation are shown in Fig. 7. For these calculations, the observed force wave forms were approximated and slightly smoothed by fitting a sixth-order polynomial. The total ground force is a maximum when the CM is at its lowest point and is a minimum when the CM is at its highest point. The amplitude of the vertical displacement is typically about 4 cm peak to peak. A similar calculation based on the horizontal force wave form indicates that the horizontal speed is a maximum when the CM is at it highest point and the speed is a minimum when the CM is at its lowest point.

A surprising result is that z is almost exactly sinusoidal, despite the fact that a is distinctly nonsinusoidal and the force wave form is asymmetric at high walking speeds. Since the a wave form is periodic, it can be modeled as a Fourier series containing a strong fundamental component and weaker harmonics. When this is integrated twice, the higher harmonic components are effectively filtered out. Conversely, the force and acceleration wave forms are very sensitive to small variations in the vertical displacement. For example, suppose that the vertical height of the CM, when it is near its maximum position, is given by $z=1-t^2/2+t^4/24$. This represents the first three terms in the series expansion of $\cos(t)$. Then z is a maximum, and $a=-2+t^2/2$ passes through a minimum, at $t=0$. However, if $z=1-t^2/2-t^4/24$ then $a=-2-t^2/2$ is a maximum at $t=0$. During the time interval $-0.3<t<0.3$, the two z wave forms are almost indistinguishable when plotted on a scale from 0 to 1 m since they are equal at $t=0$ and differ by only 0.7 mm at $t=0.3$ s. In both cases, z is a maximum at $t=0$, a is negative, and the ground reaction force F is less than Mg. However, it is unlikely that the two z wave forms could be distinguished experimentally by direct measurement, e.g., of all body segments, in an attempt to locate the position of the CM. It is known that the CM of a person is located within the pelvis, but it is not known to within one mm.

The dip in F in Fig. 6 is due to the fact that the CM is raised to its maximum height as the CM passes over the stationary foot on the ground. At this stage of the walking cycle, the CM describes an arc of radius $r\sim1$ m for adults and the ground force is given by $F=Mg-Mv^2/r$. Consequently, if v is greater than about 3.2 ms^{-1}, the ground force drops to zero and the walker becomes airborne, or breaks into a running stride.[2,3] For young children, r is smaller and they start running at a lower speed. Consequently, a child needs to run to keep up with a fast walking adult.

A simplified model of walking is obtained by regarding the leg on the ground as an inverted pendulum, and the leg in the air as a pendulum suspended at the hip. An interesting demonstration is to hold the top of a meter stick close to the hip, set it oscillating as a pendulum, and then walk in step with the pendulum. It is easy to walk faster or slower than the pendulum, but walking in step results in a comfortable walking speed and presumably helps to minimize the effort required. In general, the stride length increases with walking speed, but the cadence (i.e., step frequency) increases only slightly as the walking speed increases.[8,9]

A sideways oscillation is also observed when walking, at a frequency that is half the frequency of the vertical oscillation and slightly out of phase at low walking speeds.[4,9] At high walking speeds, the sideways oscillation is in phase with the vertical oscillation. Consequently, if one observes a walker from behind, the CM traces out a ∞-shaped Lissajous figure at low walking speeds, or a U-shaped figure at high walking speeds, with a vertical amplitude of about 4 cm and a horizontal amplitude of about 4 cm. This would also make an interesting and amusing lecture demonstration or video presentation.

V. RUNNING WAVE FORMS

Running differs from walking in that both feet are in the air for a significant part of the running cycle, there is no period when both feet are on the ground, and the feet spend a shorter fraction of the time on the ground, as well as a shorter time on the ground each stride. In walking, the feet are off the ground for about 40% of the time, so both feet are on the ground for a short period each cycle. In running, each foot is off the ground for about 70% of the time at a running speed of 5 ms^{-1}, increasing to about 80% of the time at 9 ms^{-1}. In running, the peak vertical force on each foot, while it is on the ground, is typically about $2Mg$ when running at low speed, increasing to about $3Mg$ when sprinting at a high speed. The average vertical ground force, for a complete running cycle is Mg since the average vertical acceleration is zero.[10]

Measurements of the stride length, L, of a runner at top speed indicate that $L\sim2.4$ m, where L is the distance between the landing point of the left foot and the subsequent landing point of the right foot.[8,13] This distance is larger that the distance traveled in flight, since the CM is well in front of the back foot when the runner becomes airborne, and is well behind the front foot when the front foot lands. The flight distance is typically about $0.57L$, as indicated schematically in Fig. 5.

The vertical component of the ground force acting on one foot when running at a slow pace (or jogging), measured on the homemade force plate, is shown in Fig. 8. As the foot lands on the ground, there is an initial spike due to the heel striking the ground, followed by a further increase in the ground force owing to the rapid deceleration of the CM in

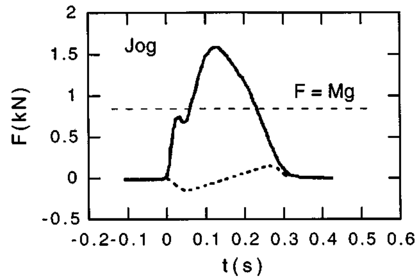

Fig. 8. The wave form observed when jogging. The dotted line represents the horizontal component of the ground force, in the direction of the gait.

the vertical direction. The ground force then decreases rapidly as the runner leaps off the ground. There is no intermediate dip in the force, below *Mg*, as there is in a walking step. The CM sinks to its lowest point while one foot is on the ground, since the knee bends significantly to absorb the initial impact and to prepare the runner for the leap off the ground. At no stage does the leg extend fully while the foot is on the ground. The CM rises to its maximum height when the feet are off the ground, not when one foot is on the ground. Consequently, the ground force wave form contains a single maximum that occurs when the vertical height of the CM is a minimum.

The horizontal component of the ground force for running, in the direction of gait, is also shown in Fig. 8. This was not measured but it is well documented in the literature. The average horizontal force is close to zero. The total horizontal force, including wind resistance, must average to zero when running at a constant average speed. The peak horizontal force is about six times smaller than the peak vertical force, and is zero when the vertical force component is a maximum, indicating that the line of action of the resultant force passes near or through the CM, and that the line of action of the resultant force makes a maximum angle of about 25° with the vertical. The horizontal force increases approximately linearly with running speed,[8] and is typically about $0.5Mg$ at a speed of 6 ms^{-1}. The average horizontal force is positive when accelerating from a starting position, and this is achieved by leaning forward so that the feet spend more time behind the CM than in front.

Analytical solutions for the vertical displacement and velocity of the CM can be obtained if the vertical force wave form is approximated as a series of half-sine pulses of the form $F=F_0\sin(\omega t)$, where F_0 is the maximum force, $\omega=\pi/\tau$, and τ is the duration of each impact on the ground. If one foot first contacts the ground at $t=0$, and the other foot first contacts the ground at $t=T$, then the acceleration of the CM in the vertical direction, for a runner of mass *M*, is given by $a=(F_0/M)\sin(\omega t)-g$ in the interval $0<t<\tau$ and $a=-g$ in the interval $\tau<t<T$. Since the average acceleration over the period $0<t<T$ is zero, F_0 is given by $F_0=(\pi T/2\tau)Mg$. The ratio T/τ is typically about 2 in running, so $F_0\sim 3Mg$. The vertical impulse, $\int_0^\tau F\,dt=MgT$ decreases as the running speed increases since T decreases, typically from $T\sim 0.37$ s at 4 ms^{-1} to about 0.25 s at 10 ms^{-1}. Wave forms computed under this approximation are shown in Fig. 9. These wave forms are the same as those for a perfectly elastic bouncing ball of mass *M* that bounces at intervals of *T* and remains in contact with the ground for a time τ each bounce. The contact time depends on the spring constant of the ball. The effective spring constant of a runner is about 2×10^4 N/m, similar to that of a tennis ball, but increases with running speed since the ground contact time decreases as the speed increases.[11]

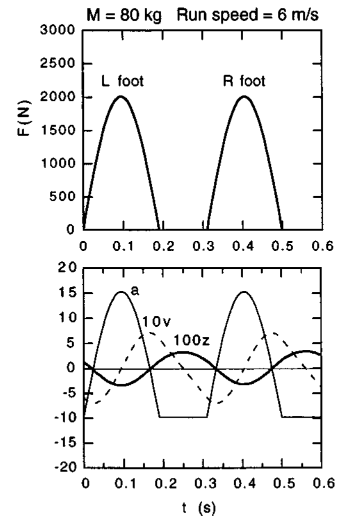

Fig. 9. A series of half-sine wave forms used to approximate the ground force when running at a speed of 6 m/s, and the corresponding values of *a*, *v*, and *z* for a runner of mass 80 kg.

An estimate of the fractional change in speed at each step can be obtained from the results shown in Fig. 8. For example, a peak horizontal force of about 200 N is observed when jogging at 3 m/s. The force lasts for about 50 ms in each direction, giving an impulse of about 5 Ns in each direction. Acting on a 70 kg jogger, the center of mass speed decreases by 0.07 m/s when the front foot pushes forward and increases by about 0.07 m/s when the back foot pushes backward. In this way, the center of mass speed is held constant to within 3%, even though the feet vary in speed by ±100%.

VI. CONCLUDING REMARKS

It has been shown that an inexpensive force plate can be constructed to provide vertical component ground force data that is similar in quality to commercial versions. Such a plate can be used in a teaching environment to demonstrate relationships between force, acceleration, velocity, and displacement in a manner that relates directly to a student's daily experiences, and in a manner that is both entertaining and informative. The plate can also be used for quantitative experiments in walking, running, and jumping, as well as other activities such as lifting weights, climbing stairs, etc. An experiment that may interest students is to test whether their highly priced sneakers have any effect on the heel strike wave form. For research work, two force plates are required, one for each foot, and sensors to detect the horizontal components of the ground force should also be incorporated. However, many universities already contain fully equipped biomechanics laboratories, and it is unlikely that ''home-made'' equipment would have a significant impact in this field. A possible improvement over commercial force plates would be to include more than four piezos between the plates in order to raise the vibration frequency and hence increase the useful frequency response. A small version designed to measure impacts of duration about 1 ms was recently described by the author.[14]

ACKNOWLEDGMENTS

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[4]J. Rose and J. G. Gamble, *Human Walking*, 2nd ed. (Williams and Wilkins, Baltimore, 1996), pp. 1–99.

[5]Available from Thomson Marconi Sonar P/L, 274 Victoria Rd., Rydalmere NSW 2116 Australia; electronic mail: jandera.zdenek@tms-pty.com.

[6]It is a common observation, when someone slips on a banana skin, that the front foot slips forward.

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[10]For the same reason, the time average weight of a 70 kg person juggling three 1 kg balls in the air is 73 kg weight.

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THE BOTTOMLESS PIT

This [godless metric] system came out of the ''Bottomless Pit.'' At that time and in the place whence this system sprang it was hell on earth. The people defied the God who made them; they worshipped the Goddess of reason. In their mad fanaticism they brought forth monsters—unclean things. Can you, the children of the Pilgrim Fathers, worship at such a shrine, and force upon your brethren the untimely monster of such an age and such a place?... Now, my friends, when the grave-diggers begin to measure our last resting places by the metric system, then understand that the curse of the Almighty may crush it just as he did the impious attempt to abolish the Sabbath.

Quoted in Frank Donovan, *Prepare Now for a Metric Future* (Weybright and Talley, New York, 1970), p. 82.