Forensic Physics 101: Falls from a height

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The physics of falling from a height, a topic that could be included in a course on forensic physics or in an undergraduate class as an example of Newton's laws, is applied to a common forensic problem.
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I. INTRODUCTION

A recent innovation in some physics departments is the introduction of courses on forensic physics.[1] Topics of interest include motor vehicle accidents,[2,3] trajectories of bullets,[4] fire and explosion investigation,[5] and materials identification and imaging methods. Another topic that could be used to illustrate a forensic application of Newton's laws is the physics of falls from a height involving serious injury or death.[6,7] Fatal falls are mostly accidental and commonly involve falling from or down objects such as a ladder, tree, stairs, balcony, or a construction site.[8–10] Falling from a height is surprisingly common, accounting for about one in eight work-related deaths. During 1997–2000, 1643 people fell off flat-roofed houses in the south-east part of Turkey.[11] Between 1937 and 1981, 720 people fell or jumped off the Golden Gate Bridge.[12]

Falls from a height are generally a safety issue, but can also be an issue for the police. Over the last five years I have investigated 15 falls involving death or serious injury for the police and the coroner in New South Wales. Most of these cases involved falls from a building or a cliff. One involved serious injury when a person was pushed off a train platform into the path of an oncoming train. All involved suspicious circumstances. Each case was different, but all involved simple applications of Newton's laws and some relevant simulations. Police and lawyers generally have a very limited understanding of Newton's laws and of experimental techniques commonly used in physics. As a result, falls that might be suspicious to a physicist are not necessarily regarded as suspicious by the police, and vice versa. There is an opportunity and a need for courses in forensic physics and for people trained in this field to be employed within the police force, the legal profession, the insurance industry, and as independent professional consultants.

Cases that are reliably witnessed or captured on film or documented with a suicide note can generally be handled by the police and the coroner or the courts, without the assistance of expert advice. In other cases, the question that needs to be answered is whether the fall was the result of an accident, suicide, or homicide. Did the victim accidently slip, trip, or overbalance, or deliberately jump or dive, or was the victim pushed or thrown? In some cases the police want to know whether a witness or a suspect has given a plausible version of the event.

The trajectory of a fall is important, but there are other aspects of a fall, such as the launch and landing phases of the fall, which can also be investigated using physical measurements and calculations. These include measurements of the speeds at which a person can run, jump, dive, be pushed or thrown, an estimate of the rotational speed associated with each launch method, measurements or estimates of runup and takeoff distances, calculations of possible trajectories based on different launch angles and wind speeds, location of the center of mass of a person on video data, and an analysis of possible bounce and impact events. When all the data are combined with other evidence obtained by the police and medical specialists, the circumstances surrounding a fall can sometimes be determined accurately. Often, there is insufficient evidence to arrive at a definite conclusion.

II. TRAJECTORY CALCULATIONS

Suppose that a person lands at a horizontal distance $d = 9$ m from a building after falling from a height $h=30$ m. The fall time is $t=\sqrt{2h/g}=2.474$ s and the horizontal launch speed is $v_x=d/t=3.64$ m/s. If the only possible launch point was a narrow ledge below an open window, could a person jump at 3.64 m/s from a standing start, dive, or be pushed or thrown through the window at that speed? To answer this question we would first need to improve on the estimate provided by the simple physics calculation. We would then need to take measurements of typical jump, dive, and throw speeds, appropriate for the estimated athletic ability of the deceased and any known suspects.

The horizontal distance traveled by a person from a given launch point to the first point of impact has three components consisting of the takeoff, flight, and landing distances. The flight distance is the horizontal distance traveled by the center of mass of the person through the air. The takeoff distance, d_T, is the horizontal distance from the center of mass to the foot on the ground at the instant that the person becomes airborne. The center of mass could be 0.5 m beyond the edge of a cliff or building when the person first becomes airborne. The landing distance, d_L, is the horizontal distance from the center of mass to the first point of impact, and may also be around 0.5 m. The horizontal launch speed required to jump a horizontal distance of 1 m is therefore zero because a person can simply step that distance at essentially zero speed.

Suppose that the center of mass of a person falls through a vertical height H and travels a horizontal distance D through the air, as shown in Fig. 1. If we ignore the effects of air resistance or wind, then H and D are related to the launch speed v_0 and the launch angle θ_0 by

$$D=\frac{v_0^2\sin(2\theta_0)}{2g}\left[1+\left(1+\frac{2gH}{v_0^2\sin^2\theta_0}\right)^{1/2}\right], \tag{1}$$

which reduces to the well known result $D=v_0^2\sin(2\theta_0)/g$ when $H=0$. The maximum horizontal range of a small projectile therefore results when $\theta_0=45°$ if $H=0$. In the long jump, the maximum range occurs when θ_0 is about 25°, partly because H is not zero but mainly because people can-

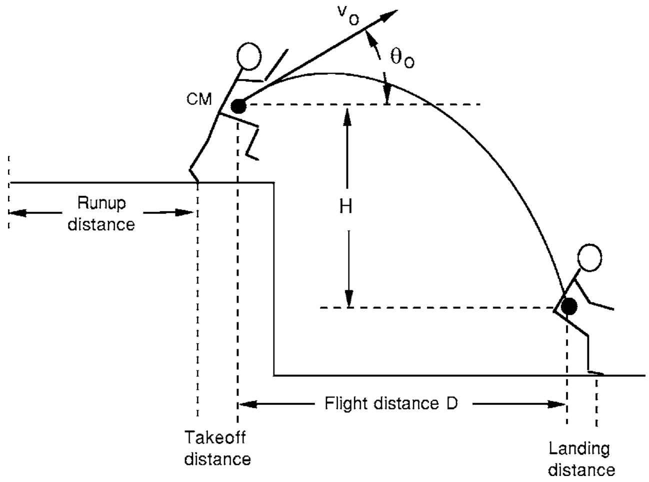

Fig. 1. A trajectory calculation needs to take into account the takeoff and landing distances, as well as the flight distance in order to estimate the required launch speed. The available runup distance, together with athletic ability, helps to determine whether such a launch is possible.

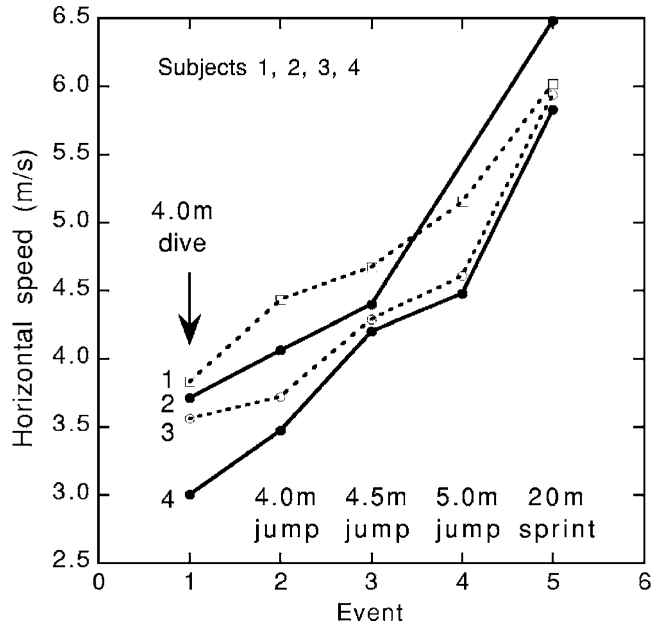

Fig. 2. Horizontal launch speeds for four adult females, with runup distances from 4.0 m to 5.0 m, and the run speed after a 20 m runup, as indicated. The 20 m run data indicates that all four females were above average in terms of athletic ability, as determined from a much larger sample of females.

not jump as fast in the vertical direction as they can run in the horizontal direction.[13–16] For a fall off a cliff or tall building, the maximum flight distance occurs at a launch angle of around 15°. The flight distance is given by the horizontal launch speed multiplied by the time in the air. If the fall height is large, then the time in the air is determined mainly by the fall height, while the horizontal launch speed is maximized by a near horizontal launch. The launch angle in any particular fall will not be known, but the flight distance is not a strong function of θ_0 for values of θ_0 in the range of practical interest. Consequently, a good estimate can be made of the horizontal launch speed, or at least of the minimum required launch speed, given that the precise launch point may also be unknown.

Air resistance results in a small correction to the result in Eq. (1), as does a headwind or tailwind. The main force on the body is that due to gravity. In addition, the air results in a drag force $F=0.5C_dA\rho v^2$, where A is the cross-sectional area of the body, $\rho=1.21\ \mathrm{kg\,m^{-3}}$ is the density of air at 20 °C, v is the speed of the body relative to the air, and C_d is the drag coefficient. The cross-sectional area depends on the orientation of the body, and C_d can be taken to be about 0.7, given that $C_d=0.5$ for a sphere and $C_d=1$ for a flat surface. For example, if $A=0.1\ \mathrm{m}^2$ and $v=20$ m/s, then $F=16.9$ N, compared with a gravitational force of 686 N on a 70 kg person. The drag force is negligible in the vertical direction for speeds up to about 30 m/s, but it can reduce the horizontal speed of a 70 kg person from say 5.0 m/s to about 4.7 m/s during a fall time of 3.0 s depending on the orientation and the horizontal component of the drag force. In the vertical orientation, with $A=0.1\ \mathrm{m}^2$ and $m=70$ kg, the terminal velocity would be 127 m/s. However, if A were increased to 0.5 m^2 by falling in a horizontal position, the terminal velocity would decrease to 57 m/s. The relevant equations of motion, including the drag force, are easy to solve numerically.[17]

III. RUN, JUMP, DIVE AND THROW SPEEDS

Running and jumping speeds are readily available for elite athletes competing in standard athletic events, but are not readily available for nonelite athletes or for a short runup preceding a run, jump, or dive. Push and throw data are not readily available either. It is also unlikely that the athletic ability of the victim will be available. Nevertheless, estimates can be obtained of the relevant abilities of a person of average athletic ability to determine whether a particular fall scenario is feasible or not. Figure 2 shows data that I obtained for a sample of four female police cadets performing various maximum effort tasks. The tasks included jumping feet first or diving head first from the edge of a swimming pool after a runup distance of 4.0 m, 4.5 m, and 5.0 m, and sprinting over a distance of 20 m on a level surface (without jumping at the end of the 20 m run). The athletic ability of the sample was then compared with that of a much larger sample of female cadets performing some of the same tasks as part of their compulsory fitness training. It was concluded that all four subjects were better than average in athletic ability but none were elite athletes. The data in Fig. 2 were obtained by filming each task and analyzing the film to determine the horizontal speed of the center of mass. A radar gun could also be used to measure running speeds. The data in Fig. 2 were used in a particular case study, as described in Sec. VII.

Other tests that I have conducted show that a person can be pushed forward at only about 1.5 m/s, because the feet remain on the ground during the push, and a strong male can throw a 60 kg female into a swimming pool at speeds up to about 4.8 m/s, depending on the technique and the strength of the male. Maximum throw speed resulted when the male had a short runup and when the female was thrown head first, and in line with her center of mass in order to minimize her rotational energy. There was no danger of the male thrower falling into the pool after a short runup because all his forward momentum was transferred to the female during the throw. By contrast, a man running toward the edge of a cliff while throwing a light weight could easily fall off the cliff himself. Two men can swing a female back and forth in a pendulum-like motion by holding her arms and feet, and the resulting throw speed is typically about 2.7 m/s, about the same as that when a pendulum falls through a height of about 0.4 m.

If a person accelerates from rest in a 100 m sprint, maximum speed is reached at about the 30–40 m mark.[18] The speed does not increase linearly with time. I found that a person can reach about 75% of maximum speed after the first 5 m. If a person then jumps forward after reaching the 5 m mark, the forward jump speed is typically about 0.5 m/s less that the corresponding run speed at the 5 m mark, because the action of planting the front jumping foot increases the backward friction force acting on the front foot. If a person dives head first at the 5 m mark, there is an additional loss of about 0.4 m/s in the forward speed due to the braking force required to generate the necessary forward rotation.

Running and jumping on an inclined surface needs to be measured separately. I found that most people can run short distances up a gradual slope at about the same speed as on a horizontal surface. However, if a person jumps forward after running up an uphill slope, there is a relatively large reduction in the forward jump speed because the normal reaction force acts backward, with a correspondingly large component in the horizontal direction. For example, six females were tested by running a distance of 20 m as fast as possible across a surface that was level for the first 15 m and sloped 5° uphill for the last 5 m. At the 20 m mark, there was a run speed reduction of 6% on average compared with a 20 m run on a level surface. When the subjects jumped forward at the 20 m mark, there was a jump speed reduction of about 15% for three of the six females, compared with the equivalent jump on a level surface. A 9% reduction was observed for the other subjects.

IV. FORWARD ROTATION

A person can stand at the top of a cliff or building and fall slowly and deliberately by pivoting forward about an axis through his/her feet. A second person can hasten the fall by pushing from behind, as shown in Fig. 3. An estimate of the launch speed can be obtained by assuming that a person in this situation behaves like a uniform, rigid rod of mass m and length L, with a center of mass at height $h=L/2$. Suppose that a horizontal force F is applied at a height d above the ground. If we assume that the rod pivots at angular velocity ω about an axis through the bottom end, then the equation of motion is given by $Fd=I_0 d\omega/dt$, where $I_0=mL^2/3$ is the moment of inertia about the axis at the bottom end. The center of mass rotates at speed $V_{\mathrm{cm}}=h\omega$. For small angles of rotation away from the vertical, $F-F_r=mdV_{\mathrm{cm}}/dt$, where F_r is the horizontal static friction force acting at the bottom end of the rod. If F_r is less than μmg, where μ is the coefficient of static friction, then the bottom end of the rod will not slide but will pivot about a fixed axis. If the person is pushed at the center of percussion, which is at a height $4h/3$ from the ground, F_r is zero.

An example will illustrate the magnitude of the parameters involved. Let $m=70$ kg and $L=1.7$ m so that $I_0=67.4$ kg m^2. Suppose that F is a constant force of 100 N of duration 0.3 s, applied at a height $d=1.4$ m. Then the rod will rotate with angular acceleration $d\omega/dt=2.08$ rad/s^2. After 0.3 s, $\omega=0.62$ rad/s, the rod rotates through an angle $\theta=5.3°$, and $V_{\mathrm{cm}}=0.53$ m/s. Because $mdV_{\mathrm{cm}}/dt=124$ N, the friction force at the bottom of the rod is 24 N, which is much less than the normal reaction force mg=686 N, so the rod will pivot provided that $\mu>0.035$.

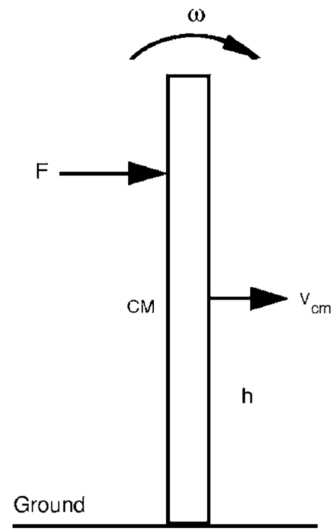

Fig. 3. When a person is pushed from behind, the horizontal launch speed is typically only about 1.5 m/s. To avoid injury to volunteers the essential physics can be extracted by pushing a heavy object.

The subsequent behavior of the rod can be estimated by assuming that the rod continues to pivot until it reaches a horizontal position. However, the bottom end of the rod will commence to slide forward as the rod nears the horizontal position, as described in Ref. 19, resulting in a finite horizontal launch speed. For the given parameters and $\mu=0.7$, the horizontal launch speed is 1.24 m/s without a push, and the vertical speed is 3.40 m/s when the rod reaches a horizontal position. The effect of a 100 N push lasting 0.3 s is to increase the horizontal launch speed to 1.26 m/s. The increase in the launch speed is negligible because the additional kinetic energy generated by the push force is negligible compared with the initial potential energy. The horizontal launch speed for a 200 N force applied for 0.3 s increases to only 1.34 m/s, and the vertical launch speed increases to only 3.42 m/s.

Forward rotation during a dive can be estimated in a manner similar to that for a push. In a running dive a person runs toward the launch point and then applies a horizontal breaking force, F_r, by pushing forward on the ground with the front foot or with both feet. In a normal running stride the front foot also pushes forward on the ground and therefore generates a breaking force. In a dive or forward somersault the front foot pushes more firmly so that the person can simultaneously gain extra height and rotate forward. In that case the applied torque is given by $F_r h=I_{\mathrm{cm}} d\omega/dt$, where $I_{\mathrm{cm}}=mh^2/3$ and $F_r=-mdV_{\mathrm{cm}}/dt$. The change in horizontal speed, ΔV_{cm}, is therefore related to the change in angular speed, $\Delta\omega$, by $\Delta V_{\mathrm{cm}}=-h\Delta\omega/3$. For example, if a person rotates from an approximately upright position through 45° during a 0.5 s diving action, then $\Delta\omega=1.57$ rad/s and $\Delta V_{\mathrm{cm}}=-0.44$ m/s if $h=0.85$ m. This result is consistent with the measurements shown in Fig. 2, where maximum effort dive speeds are typically about 0.4 m/s less than maximum effort feet-first jump speeds.

V. STANDING JUMP

A person jumping or diving off a tall building or a cliff can do so in many different ways. However, if the only launch platform available is relatively narrow or bounded by a fence nearby, then a standing jump or dive may be the only option. A wider ledge might allow for one or two steps before jumping or diving. In that case, a typical horizontal launch speed for a person of average athletic ability would be 2–3 m/s. A good swimmer can dive into a swimming pool at about 4 m/s by pushing horizontally against a vertical surface such as the starting block. Each of these speeds is larger than the speed at which a person can be pushed, so it is theoretically possible to distinguish between a push and a jump or dive under these conditions. However, a launch speed of 2 m/s or less does not allow for such a distinction, at least in terms of the estimated launch speed.

A possible exception would be a launch speed less than about 0.5 m/s. A person jumping at minimum speed or rotating forward off a ledge will be projected off the ledge at a horizontal speed of about 1 m/s due to the horizontal component of the force acting at his or her feet. A simple demonstration of this effect is to slide a block of wood at very low speed off the edge of a table. The block does not fall vertically when its center of mass extends beyond the edge of the table. Rather, the block rotates and is then projected outward at finite horizontal speed, landing on the floor at a point well past the edge of the table.

The optimum takeoff angle in the standing long jump has been calculated[20] as about 20°–25°. This calculation assumes that the jump starts and ends on the same horizontal surface. As mentioned, the optimum takeoff angle is about 15° when jumping from a large height. The physics of jumping in a vertical direction has been described by Linthorne.[21]

VI. LOCATION OF THE CENTER OF MASS

A measurement of the horizontal launch speed and launch angle of a person jumping or diving into a swimming pool can be obtained by filming the event with a video camera. The horizontal and vertical distance needs to be calibrated by filming an object of known length located in the same plane, while the time scale is determined by the frame rate, typically 25 or 30 frames/s. The object is to determine the launch speed and angle of the center of mass. If the jumper or diver maintains the same orientation through the air, the speed of the center of mass would be the same as that of any other part of the body, and there would be no need to locate the center of mass. However, a person jumping or diving usually moves his or her arms and legs into different positions during the flight phase, in which case an estimate of the location of the center mass needs to be made for each frame. Depending on the desired accuracy of the measurement, the location of the center of mass can be estimated by eye or calculated from the measured orientation of each body segment and an estimate of the fractional body mass of each segment.

The center of mass of a person standing upright is located near a point between the hips. By lifting one leg or one arm, the center of mass is raised by only 1 or 2 cm because the mass of one arm or one leg is only a small fraction of the total body mass. Consequently, we can usually visually estimate the position of the center of mass to within 1 or 2 cm. Otherwise, the position of the center of mass of each body segment needs to be estimated, in which case the (x,y) coordinates of the center of mass of the whole body can be found as a weighted means.[22]

VII. TWO CASE STUDIES

I have previously described two real-life examples of falls from a height.[6] In one case a woman died as a result of a fall at night from a third floor balcony. Her partner was inside, but saw her fall head first into the darkness after warning her that she was leaning too far forward. The police arrived soon after and noticed that she had left an imprint of her whole upper body on the outside of a vertical glass panel forming part of the balcony wall. The imprint showed that the woman was upright when she fell. Furthermore, she landed feet first but then rotated backward, striking her head on the pavement below. The police were suspicious of her partner's statements, and contacted me for an opinion. A closer inspection of the glass panel showed that there were two facial imprints, including an upside-down impression at the bottom of the panel. The woman was still holding onto the balcony rail with both hands when she struck the bottom of the panel, but managed to swing around into an upright position, with one hand on the rail, and struck the top part of the panel a second time. In this case, the trajectory of the fall was irrelevant because sufficient evidence was available on the glass panel to reconstruct events consistent with her partner's description. One of my students was able to repeat the sequence of events on the balcony, in slow motion and in the safe environment of the University gym.

The second case involved the fatal fall of a slim woman from a 30-m-high cliff at a notorious suicide spot. There appeared to be no suspicious circumstances, apart from the fact the woman landed about 12 m from the vertical cliff face. The police noted that the distance was unusually large but did not pursue the matter. Several years later I was asked for an opinion, and I then conducted the tests shown in Fig. 2, based on the fact that the available run-up distance from the safety fence to the edge of the cliff was 4 m at most. Given that the required horizontal launch speed was at least 4.5 m/s and the woman had no special athletic ability, further tests were conducted which indicated that she was thrown head first by a strong male.

VIII. CONCLUSION

The cause of an unwitnessed fall from a height, resulting in serious injury or death, is often difficult to determine. The physics of the fall can sometimes lead to a solution or at least help to eliminate some of the possible causes. The problem is sufficiently common that the topic warrants inclusion in courses dealing with forensic physics. In some universities forensic physics is offered as part of a postgraduate diploma, and other universities offer forensic physics as part of a range of courses leading to an undergraduate degree in forensic science. Most of the physics issues concerning falling accidents involve elementary mechanics and straightforward experiments, or experimental simulations of collision events using objects rather than people, in which case the subject matter is suitable for inclusion in undergraduate or postgraduate courses, depending on the course structure.

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[1]Outlines of courses in forensic physics can be found at ⟨uoit.ca⟩, ⟨www.kent.ac.uk/⟩, ⟨ntu.ac.uk⟩, ⟨sussex.ac.uk⟩, and ⟨cit.act.edu.au⟩ by searching on undergraduate courses for forensic science.
[2]R. W. Rivers, *Basic Physics: Notes for Traffic Crash Investigators and Reconstructionists* (C. Thomas, Springfield, IL, 2004).
[3]P. K. Tao, *The Physics of Traffic Accident Investigation* (Oxford University Press, Oxford, 1987).
[4]T. A. Warlow, *Firearms, the Law and Forensic Ballistics* (Taylor and Francis, London, 2004), 2nd ed.
[5]D. J. Icove and J. D. DeHaan, *Forensic Fire Scene Reconstruction* (Pearson Prentice–Hall, Upper Saddle River, NJ, 2004).
[6]R. Cross, "Fatal falls from a height: Two case studies," J. Forensic Sci. **51**, 93–99 (2006).
[7]K.-P. Shaw and S. Y. Hsu, "Horizontal distance and height determining falling pattern," J. Forensic Sci. **43**, 765–771 (1998).
[8]L. Li and J. E. Smialek, "The investigation of fatal falls and jumps from heights in Maryland (1987–1992)," Am. J. Forensic Med. Pathol. **15**, 295–299 (1994).
[9]R. Bux, M. Parzeller, and H. Bratzke, "Causes and circumstances of fatal falls downstairs," Forensic Sci. Int. **171**, 122–126 (2007).
[10]K. Toro, N. Szlavik, A. Meszaros, G. Dunay, M. Soos, and E. Keller, "Jumping and falling death in children, adolescents, and young adults," J. Clinical Forensic Medicine **13**, 129–134 (2006).
[11]Y. Yagmur, C. Guloglu, M. Aldemir, and M. Orak, "Falls from flat-roofed houses: A surgical experience of 1643 patients," Injury **35**, 425–428 (2004).
[12]G. Lukas, J. Hutton, R. Lim, and C. Mathewson, "Injuries sustained from high velocity impact with water: An experience from the Golden Gate Bridge," J. Trauma **21**, 612–618 (1981).
[13]N. P. Linthorne, M. S. Guzman, and L. A. Bridgett, "Optimum take-off angle in the long jump," J. Sports Sci., **23**, 703–712 (2005).
[14]L. A. Bridgett and N. P. Linthorne, "Changes in long jump take-off technique with increasing run-up speed," J. Sports Sci. **24**, 889–897 (2006).
[15]A. Rex, S. Hanzely, and J. Barrow, "Is a good long jumper a good high jumper?," Am. J. Phys. **69**, 104–106 (2001).
[16]O. Helene and M. T. Yamashita, "A unified model for the long and high jump," Am. J. Phys. **73**, 906–908 (2005).
[17]H. Brody, R. Cross, and C. Lindsey, *The Physics and Technology of Tennis* (Racquet Tech, Solana Beach, CA, 2002).
[18]G. Wagner, "The 100-meter dash: Theory and experiment," Phys. Teach. **36**, 144–146 (1998).
[19]R. Cross, "Fall and bounce of pencils and other elongated objects," Am. J. Phys. **74**, 26–30 (2006).
[20]M. Wakai and N. P. Linthorne, "Optimum take-off angle in the standing long jump," Hum. Mov. Sci. **24**, 81–96 (2005).
[21]N. P. Linthorne, "Analysis of standing vertical jumps using a force platform," Am. J. Phys. **69**, 1198–1204 (2001).
[22]M. Cluss, K. Laws, N. Martin, T. Scott Nowicki, and A. Mira, "The indirect measurement of biomechanical forces in the moving human body," Am. J. Phys. **74**, 102–108 (2006).