DOING PHYSICS WITH MATLAB

MATHEMATICAL ROUTINES

THE UNNORMALIZED SINC FUNCTION

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math_sinc_function.m
mscript used to investigate the sinc function. The mscript is divided into a number of cells that should be run independently by hitting the **Ctrl** and **Enter** keys together.

> **simpson1d.m**
> Function to give the integral of a function using Simpson's 1/3 rule.
>
> **turningPoints.m**
> Function to find the zero crossings of a function and its maxima and minima.

THE UNNORMALIZED SINC FUNCTION

The sinc function is widely used in optics and in signal processing, a field which includes sound recording and radio transmission.

In mathematics, physics and engineering, the **unnormalized cardinal sine function** or **sinc function**, denoted by **sinc(*x*)** is defined by

$$y(x) = \frac{\sin(x)}{x}$$

At $x = 0$ the sinc function has a value of 1.

$$y(0) = \frac{\sin(0)}{0} = 1$$

Figure (1) shows a plot of the sinc function.

The sinc function is the **zeroth order spherical Bessel function of the first kind**

$$j_0(x) = \frac{\sin(x)}{x}$$

All the zero of the sinc function occur at non-zero integer multiples of π

$$y(m\pi) = 0 \qquad m = \pm 1, \pm 2, \pm 3, \ldots$$

Hence, the zeros of the sinc function are evenly spaced, the spacing being equal to π as shown in figure (2).

The local maxima and minima of the sinc function correspond to its intersections with the cosine function.

$$y(x_m)\frac{\sin(x_m)}{x_m} = \cos(x_m) \qquad \text{maxima and minima}$$

where the derivative of sin(x)/x is zero and thus a local extremum is reached as shown in figure (2).

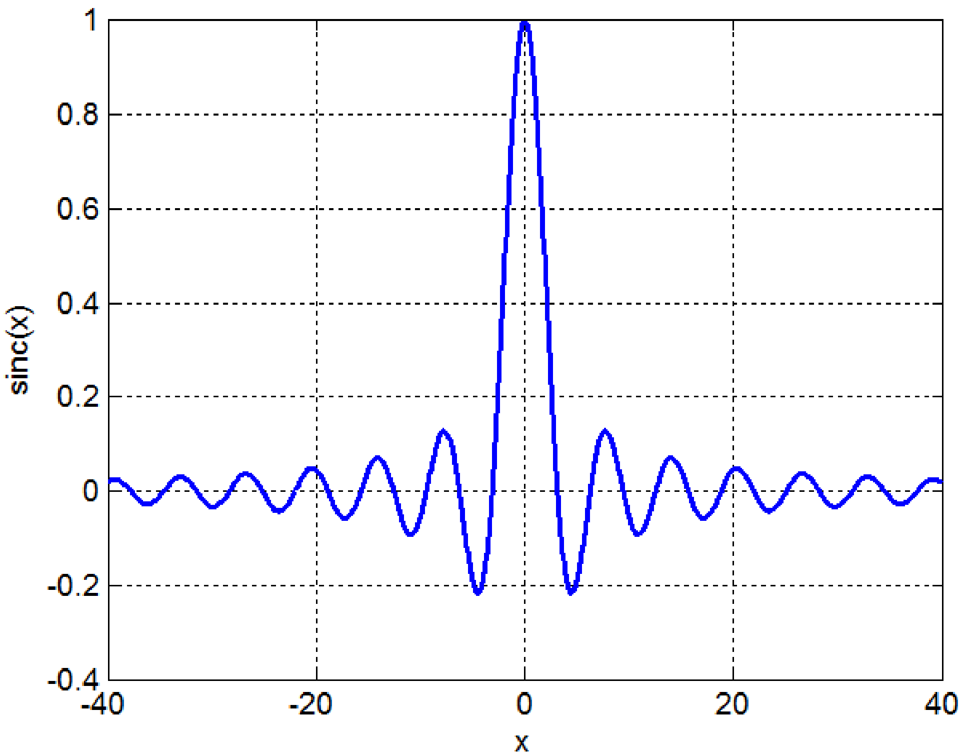

Fig. 1. The unnormalized sinc function $y(x) = \frac{\sin(x)}{x}$ plotted against x.

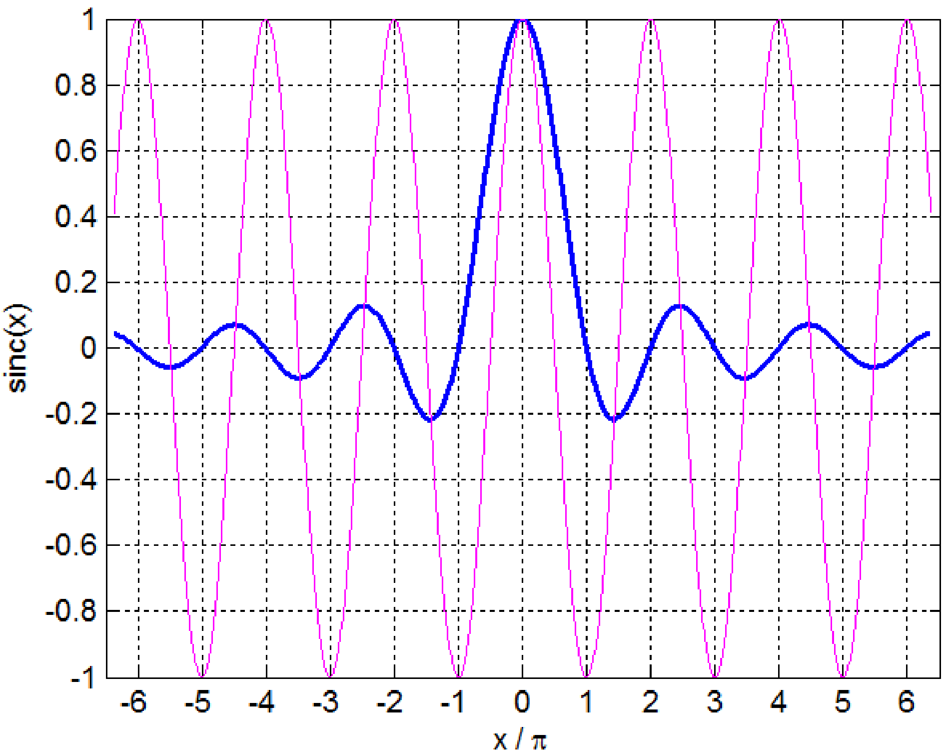

Fig. 2. The unnormalized sinc function $y(x) = \dfrac{\sin(x)}{x}$ plotted against x/π.

The zeros occur at $\left(\dfrac{x}{\pi}\right) = \pm 1, \pm 2, \pm 3, \ldots$. The magenta curve is the cosine function cos(x).

Table 1. x/π values for the max and min values of $y_m = \sin x / x$ and $y_m{}^2 = (\sin x / x)^2$ as shown in figures (2) and (3).

x / π	0	± 1.429	± 2.462	± 3.470	± 4.478	± 5.486
y_m	1.000	- 0.2172	0.1284	- 0.0913	0. 0709	- 0.0580
$y_m{}^2$	1.000	0.0472	0.0165	0.0083	0.0050	0.0034

The values for the zero crossings and minima and maxima were found using the function **turningPoints.m** .

[View document of Turning points of a function]

The square of the sinc function $(\sin x / x)^2$ gives the intensity distribution on a screen for the Fraunhoffer diffraction for a single slit.

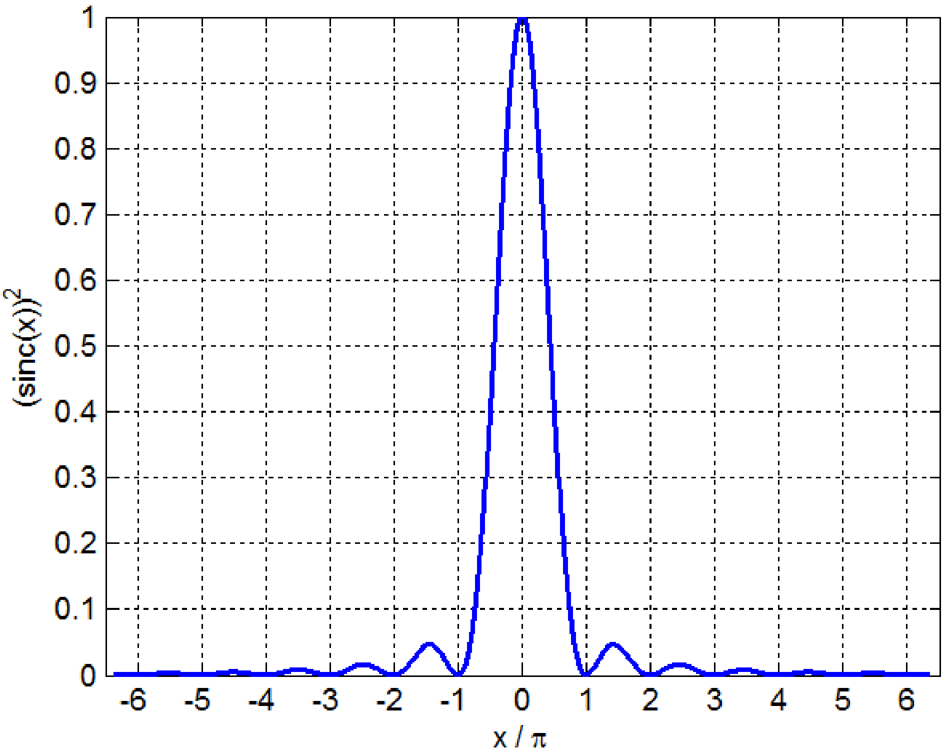

Fig. 3. The $y(x) = \left(\frac{\sin(x)}{x}\right)^2$ plotted against x/π.

The **gradient** of the sinc function can be found using the Matlab gradient command

```
xMin = -20;                          % range for x values
xMax = 20;
N = 999;
x = linspace(xMin, xMax, N);         % x values
y = sin(x+eps) ./(x+eps);            % y values   sinc(x)
yC = cos(x);                         % cosine function
dy_dx = gradient(y);                 % gradient of sinc function
```

The integral of the sinc function is

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin(x)}{x}\,dx = 1$$

This integral can be computed using the function **simpson1d.m**. In executing the function **simpson1d.m** the limits of the integral and the number of partitions can be increased until the answer converges.

```
%%  integral of sinc and (sinc)^2

clear all
close all
clc

xMin = -1500;              % range for x values
xMax = 1500;
N = 99999;               % number of partitons
x = linspace(xMin, xMax, N);    % x values
y = sin(x+eps) ./(x+eps);        % y values   sinc(x)
integral = simpson1d(y,xMin,xMax)/pi
```

xMin = -1500	xMax = +1500	N = 9999	integral = 1.0000
xMin = -1000	xMax = +1000	N = 999	integral = 0.9996
xMin = -200	xMax = +200	N = 99	integral = 1.6703
xMin = -200	xMax = +200	N = 999	integral = 0.9985

View document of Simpson's 1/3 rule