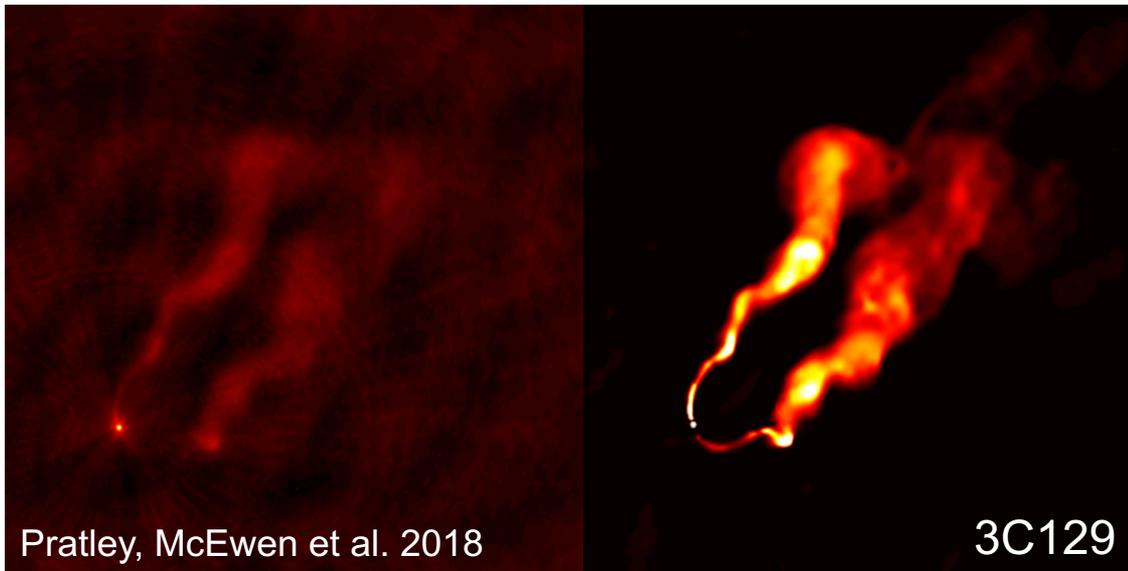


# Sparse image reconstruction of radio interferometric observations using PURIFY



Luke Pratley, Jason D. McEwen, Xiaohao Cai,  
Mayeul d'Avezac (UCL)

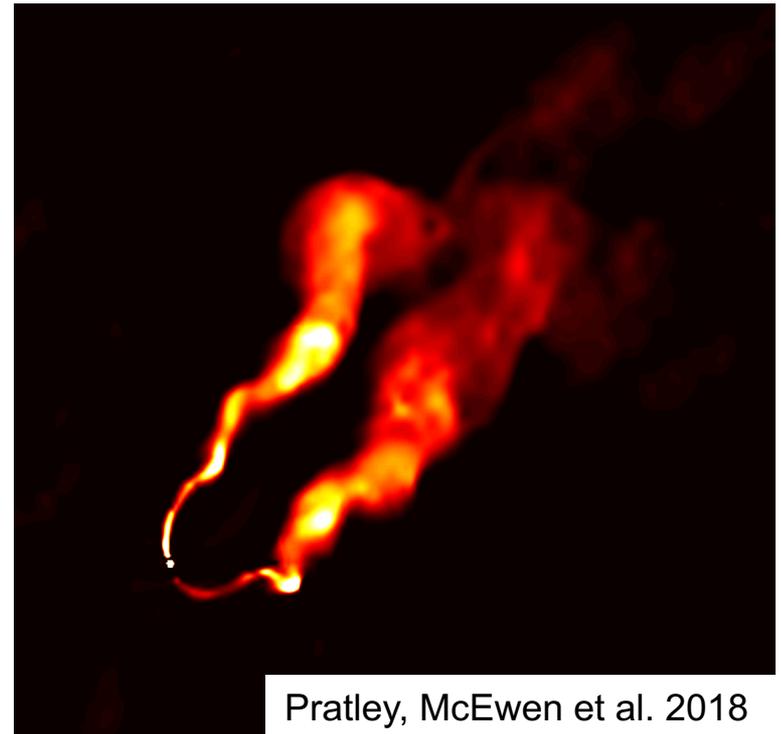
Rafael E. Carrillo, Alexandru Onose, Yves Wiaux (HWU)

# Outline

- PURIFY: Real Data
- Error Maps and Hypothesis Tests
- Online Imaging
- All Sky Imaging

## PURIFY: State-of-the-art Imager

- Next-generation imager (beyond CLEAN)
- Statistical-Foundations (critical for science)
- State-of-the-art algorithms (fast, distributed, multi-threading)
- Naturally built for polarization (Pratley & Johnston-Hollitt, 2016)



# PURIFY/SOPT

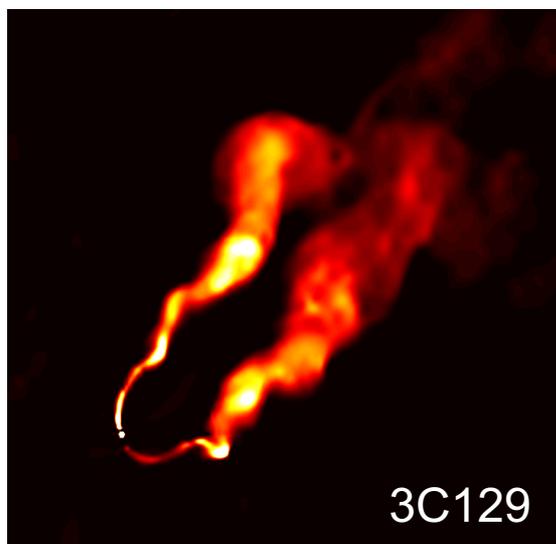
- PURIFY can be found at <http://basp-group.github.io/purify>
- Developed with UCL research computing
- It uses sparse optimisation solvers from SOPT <http://basp-group.github.io/sopt>  
(developed with PURIFY)

# Distributed Algorithms

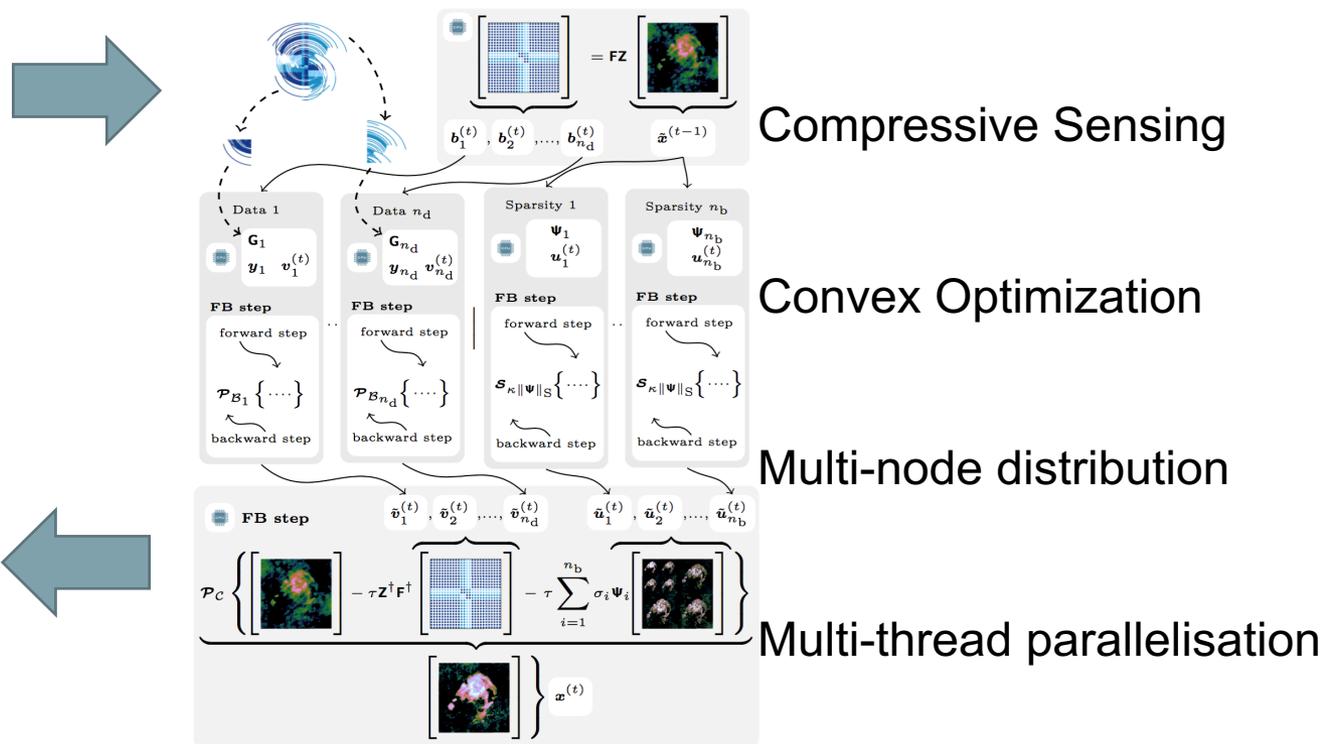
Observation



Final Image



3C129



Distributed Algorithms

Onose et al, 2016

Carrillo, McEwen, Wiaux, 2014

# PURIFY: Sparsity

Wavelet Operator

$$\min_{\mathbf{x} \in \mathbb{R}^N} \|\Psi^\dagger \mathbf{x}\|_{\ell_1}$$

Reconstruction  
(Model of the sky)

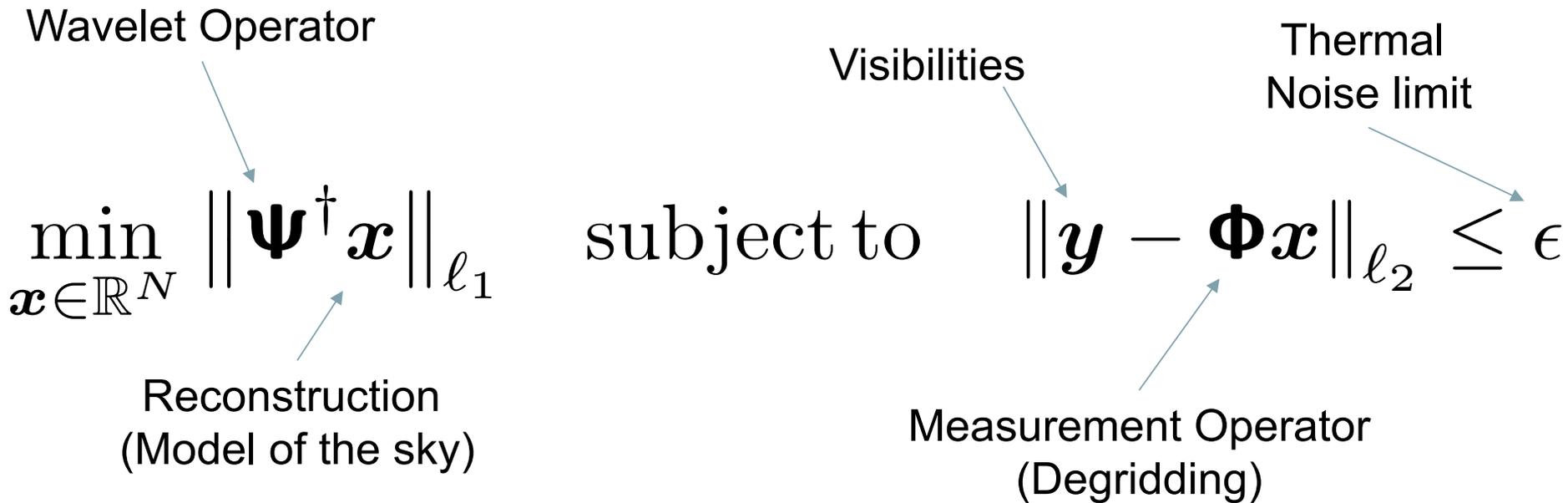
subject to

Visibilities

$$\|\mathbf{y} - \Phi \mathbf{x}\|_{\ell_2} \leq \epsilon$$

Measurement Operator  
(Degriding)

Thermal Noise limit



- Model the sky using a minimal (sparse) amount of CLEAN components. (Occam's Razor)
- Uses wavelets for compact/extended structures.
- Works for simulations and real data.

# PKS 0114-476 (ATCA)

Standard CLEAN

PURIFY

Natural

Uniform

Model

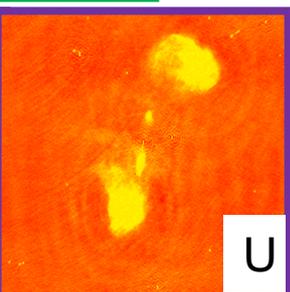
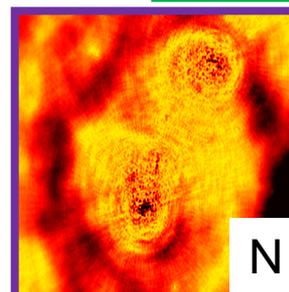
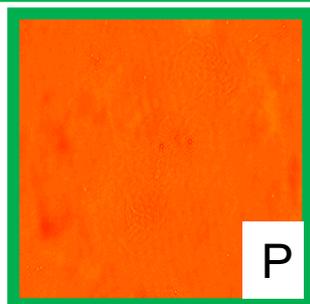
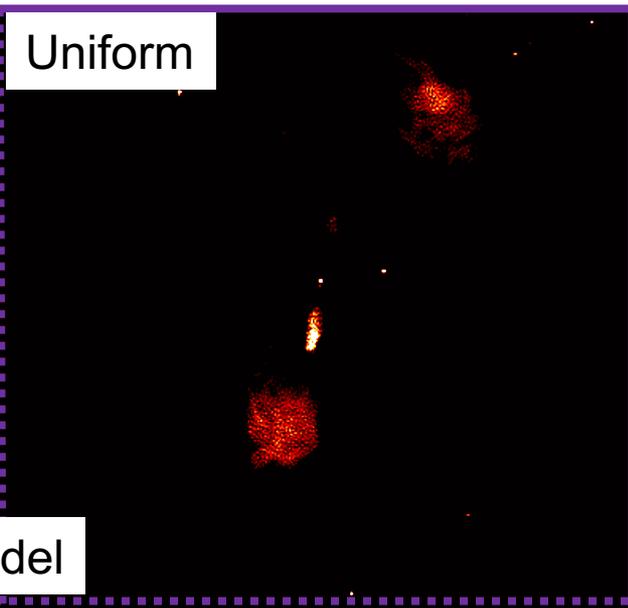
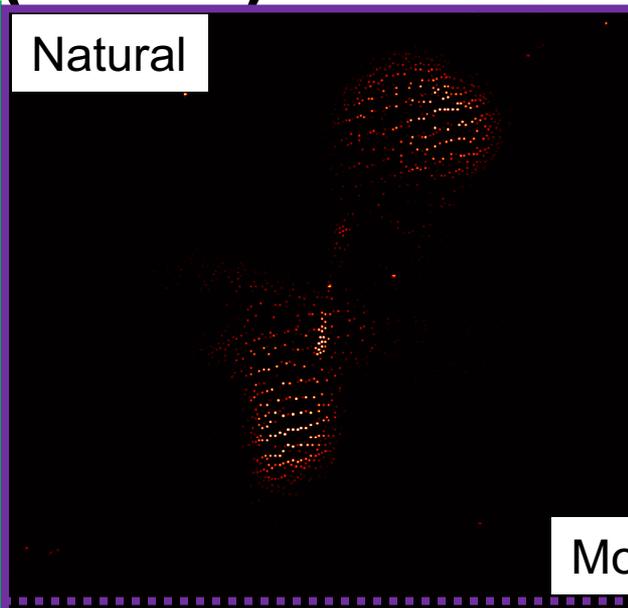
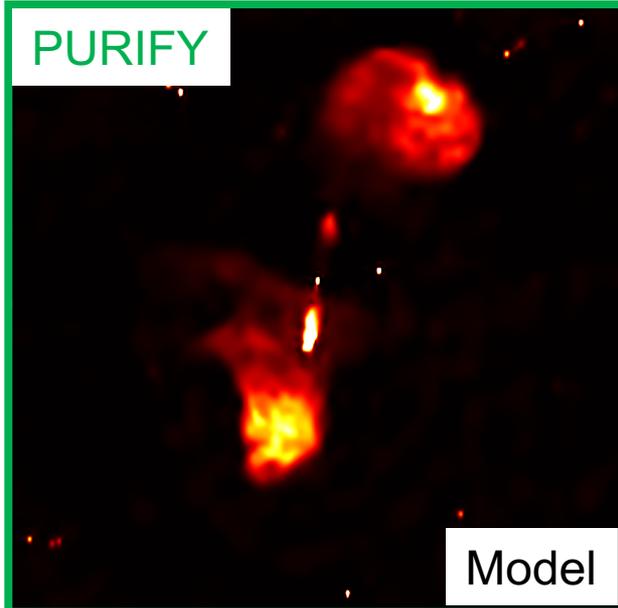
Model

Restored

P

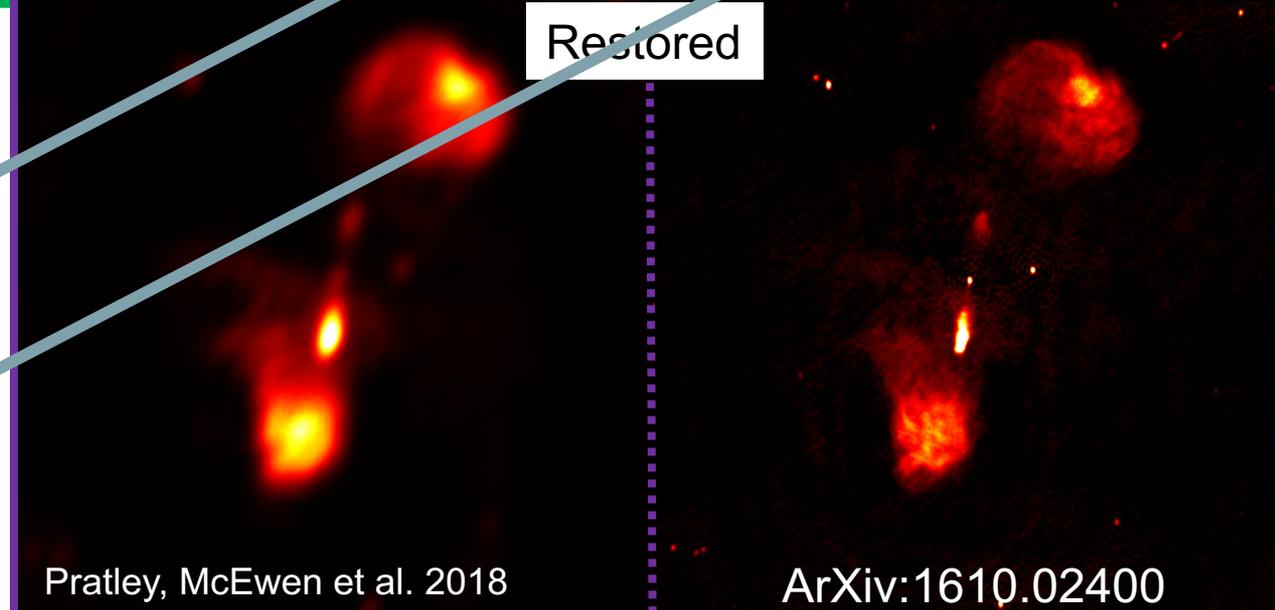
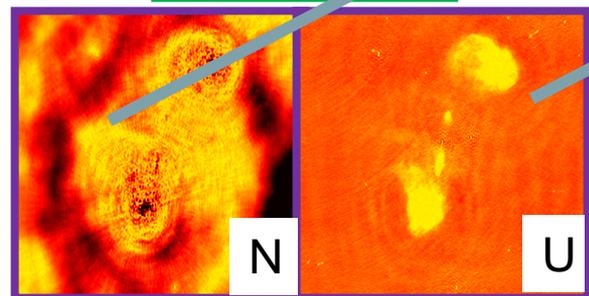
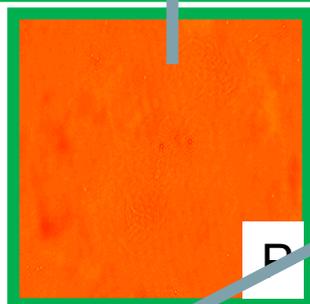
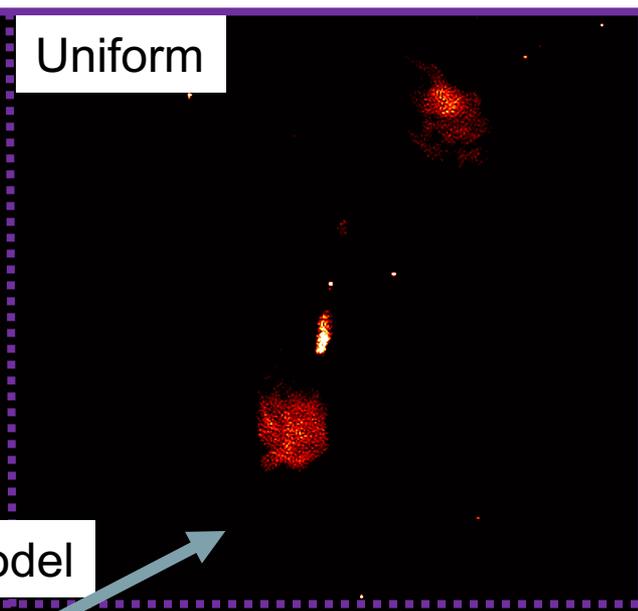
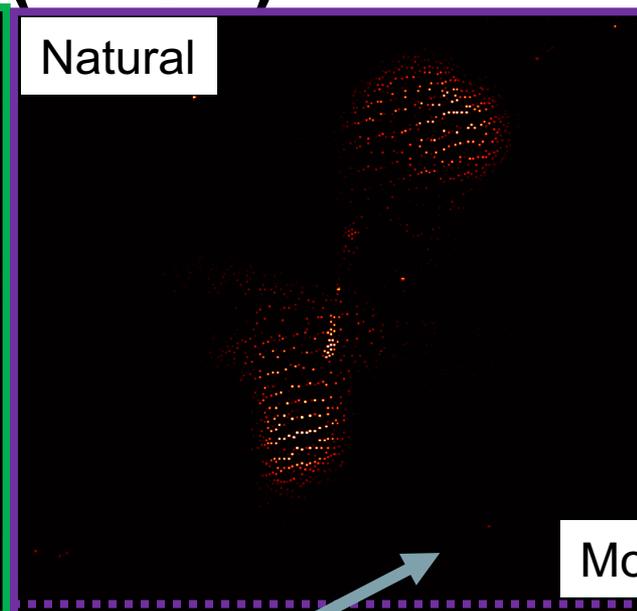
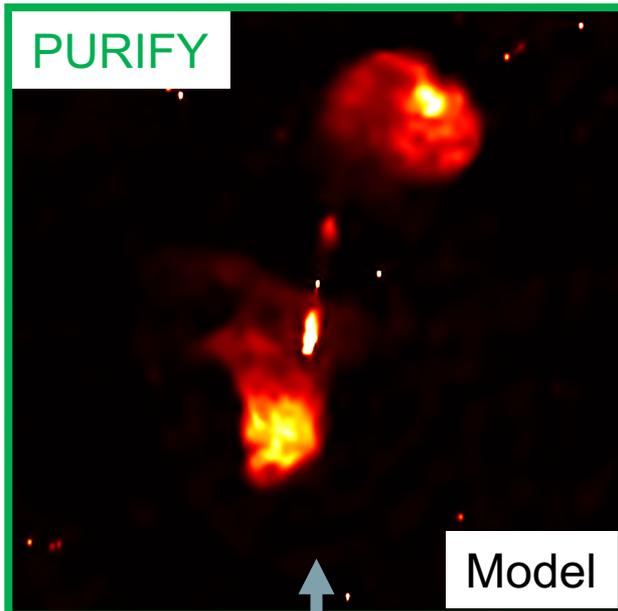
N

U



# PKS 0114-476 (ATCA)

Standard CLEAN



## Uncertainties: MAP Estimation

- Most likely model given data

$$p(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})}{\int_{\mathbb{R}^N} p(\mathbf{y}|\mathbf{s})p(\mathbf{s})d\mathbf{s}}$$

$$\mathbf{x}_{\text{MAP}} = \underset{\mathbf{x}}{\operatorname{argmin}} [-\log p(\mathbf{x}|\mathbf{y})]$$

- Gaussian error on visibilities, sparsity on model

$$\mathbf{x}_{\text{MAP}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left[ \mu \|\boldsymbol{\Psi}^\dagger \mathbf{x}\|_1^2 + \|\mathbf{y} - \boldsymbol{\Phi} \mathbf{x}\|_2^2 / 2\sigma^2 \right]$$

(Analysis)

- Forward Backward Algorithm

$$\mathbf{a}^{(i+1)} = \operatorname{soft}_{\lambda^{(i)}\mu} \left( \mathbf{a}^{(i)} - \lambda^{(i)} \boldsymbol{\Psi}^\dagger \boldsymbol{\Phi}^\dagger (\boldsymbol{\Phi} \boldsymbol{\Psi} \mathbf{a}^{(i)} - \mathbf{y}) / \sigma^2 \right)$$

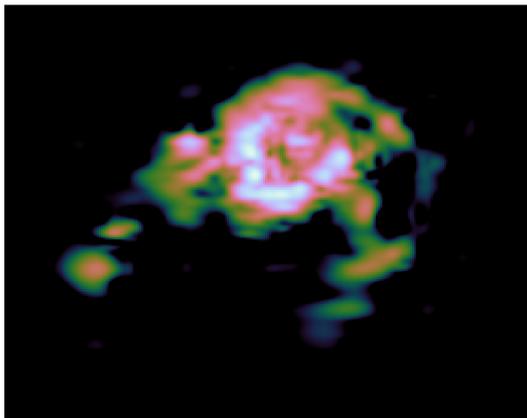
$$\mathbf{x}^{(i+1)} = \boldsymbol{\Psi} \mathbf{a}^{(i+1)}$$

(Synthesis)

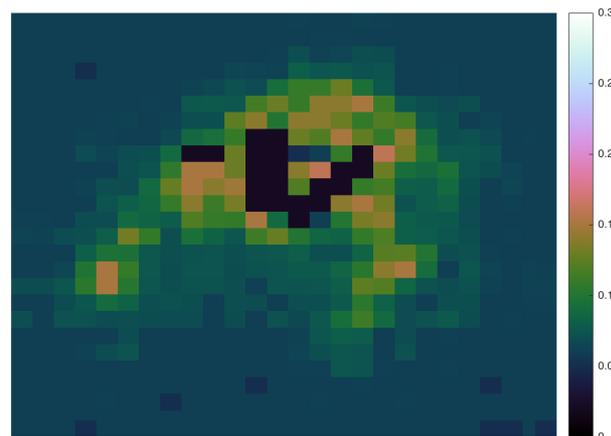
# Credible Interval: Error Bars

- Want uncertainty on region in image
- Use MCMC? Very slow!  
(Cai, McEwen, Peryra 2017a, arXiv:1711.04818)
- Alternative: Find the Credible Interval (50,000x faster!)

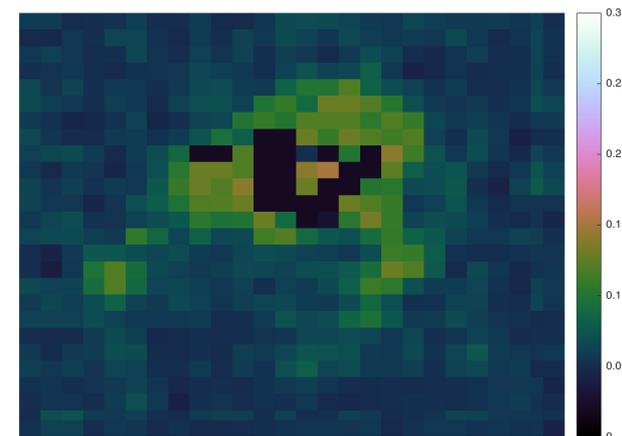
MAP Estimation



Credible Region



Proximal MCMC



# Hypothesis Testing

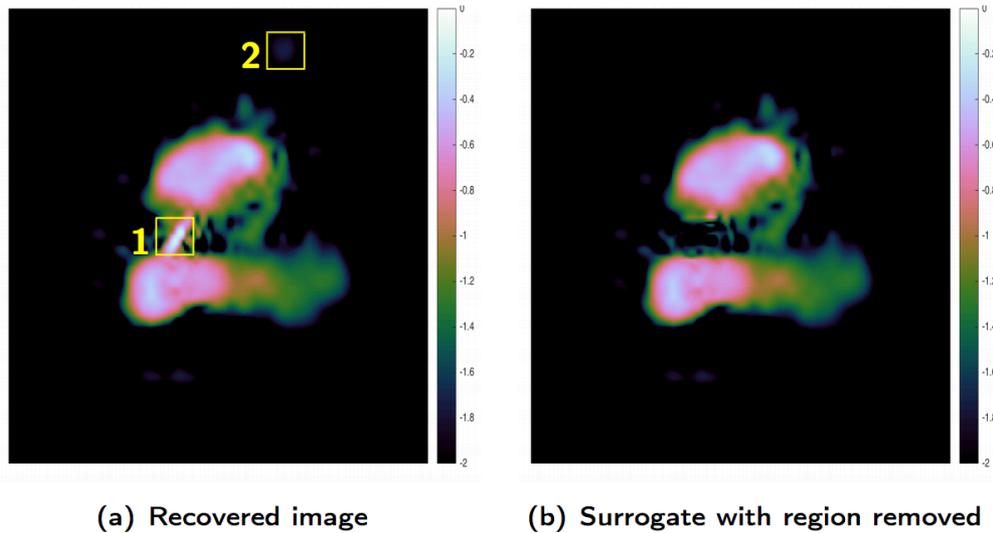
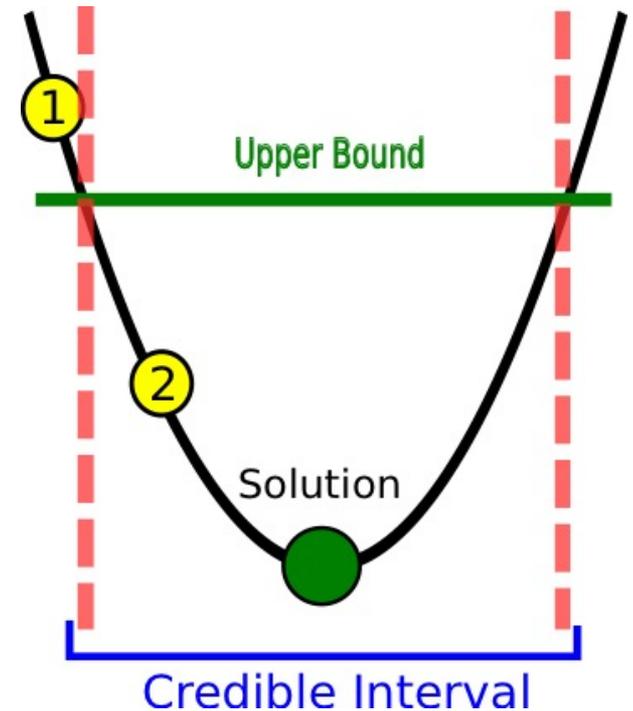


Figure: 3C288



Remove 1 – outside error – Region 1 unphysical is unlikely

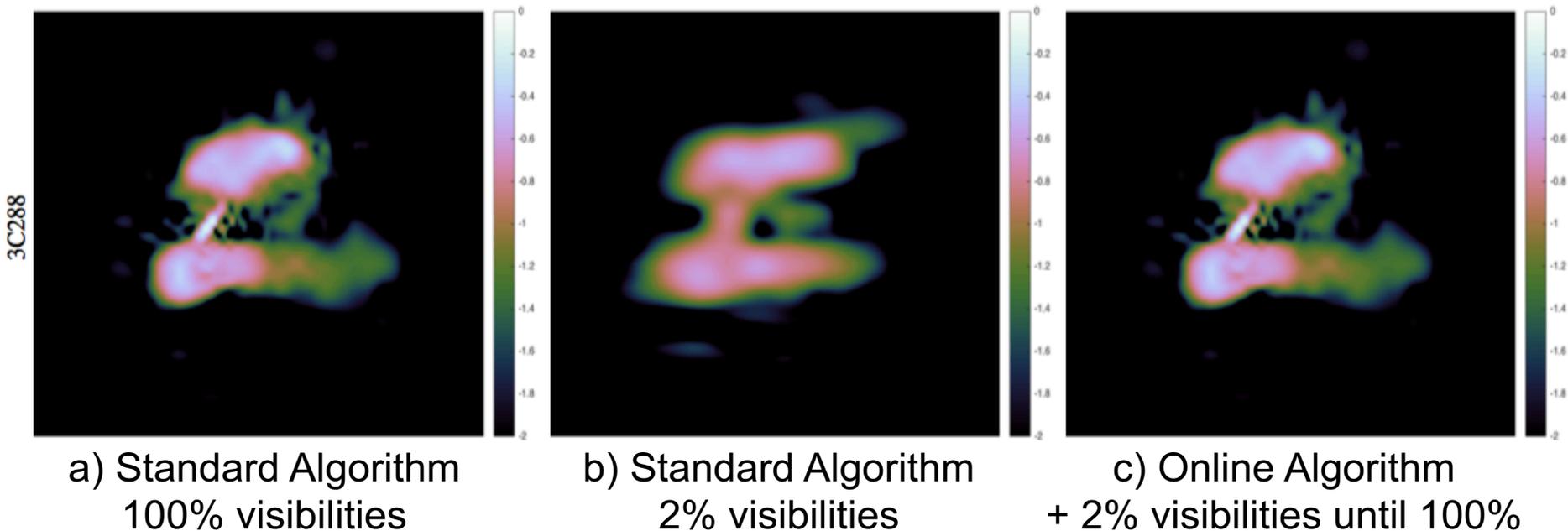
Remove 2 – within error – Region 2 unphysical is possible

# Online Imaging

Released on ArXiv Today!  
ArXiv:1712.04462

- Online Imaging – Reconstruct image while streaming in data.
- Uses less computation and memory than offline method (but same quality!)

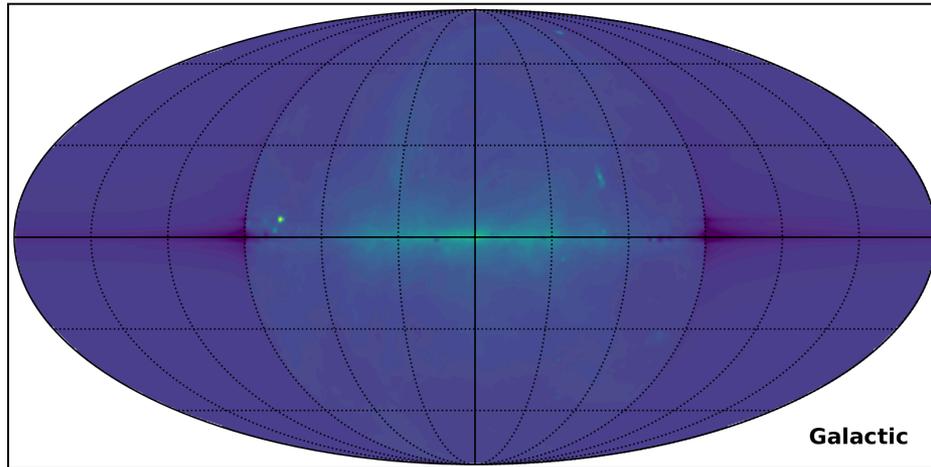
Assimilate and Discard!



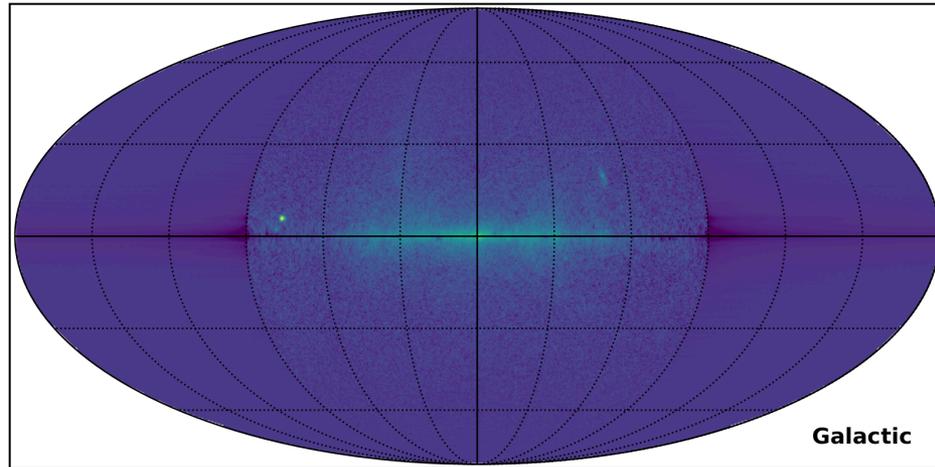
# All Sky Imaging/Reconstruction

- Image efficiently on the sphere

Ground Truth



Simulated Dirty Image



Pratley & McEwen (in prep.)

McEwen & Scaife, 2008

Shaw et al. 2014

Eastwood et al. 2017

Carozzi 2015

- Perform Sparse reconstruction on the sphere.

See Wallis, Wiaux, McEwen et al. 2017, ArXiv:1608.00553

## Summary

- Sparse image reconstruction works on simulations and real data
- Distributed image reconstruction
- Uncertainty Quantification
- Online Imaging (reduce memory and computation)
- All-sky modelling and imaging