On the Variance of Calibration Solutions:

Quality-based Weighting Schemes

Etienne Bonnassieux, 3rd year PhD student

Supervisors: Cyril Tasse, Oleg Smirnov, Philippe Zarka
Work in collaboration with: Landman Bester, Trienko Gobler, ...

1. Scientific Context: SKA and its Pathfinders

Slide credit: Cyril Tasse

Collecting area: 1 sq. km

Resolution: ~10 mas a 1 GHz
(a 1 euro coin at 400 kilometers)
Sensitivity: ~50 nJy/Beam
[8 hours, 500Mhz bandwidth]

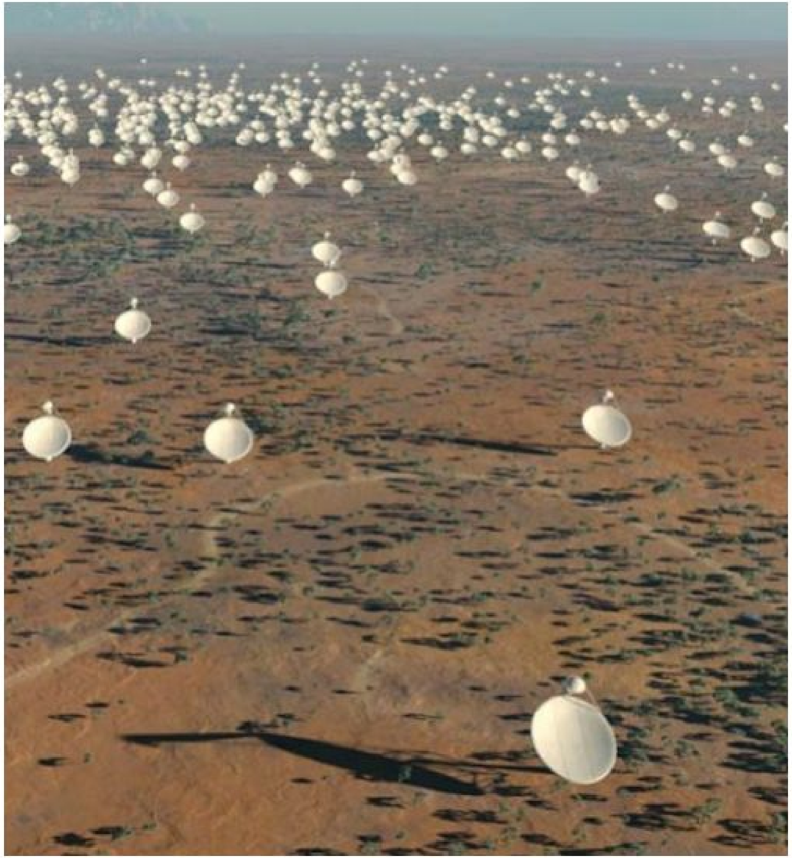
Field of view: ~ 1 degré carré

360.000x360.000 pixels images

Survey speed: x10.000

A few huge radiotelescopes prototypes of the SKA:

- **MeerKAT (under construction)**
- **LOFAR (operational)**
- ASKAP
- .....



2. New era, new challenges

Key challenges for new era of radio interferometry. Importantly:

- ➔ SKAta volume…
  - ◆ 100 times global internet traffic!!!!
  - ◆ Need on-the-fly calibration + imaging
  - ◆ Can only realistically store final science products (images)
- ➔ Need fast, efficient algorithms to improve final images.

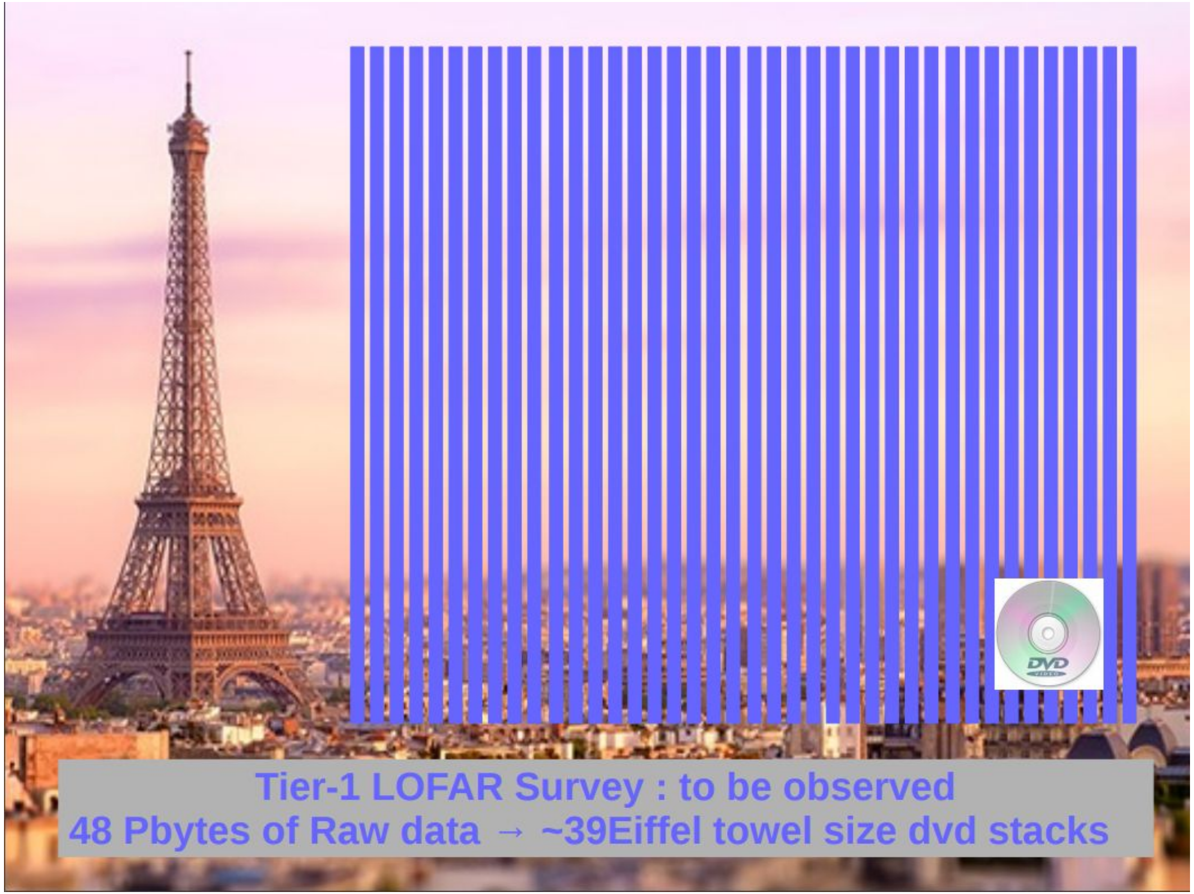
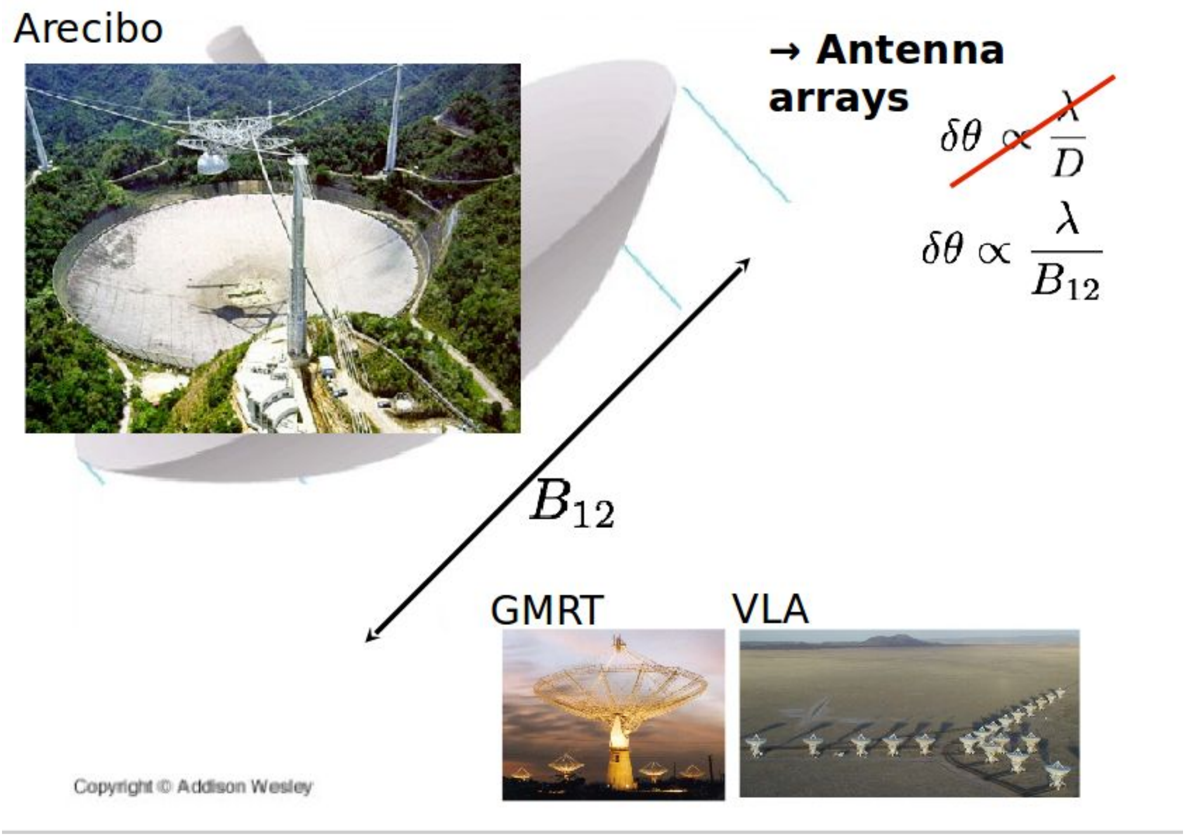


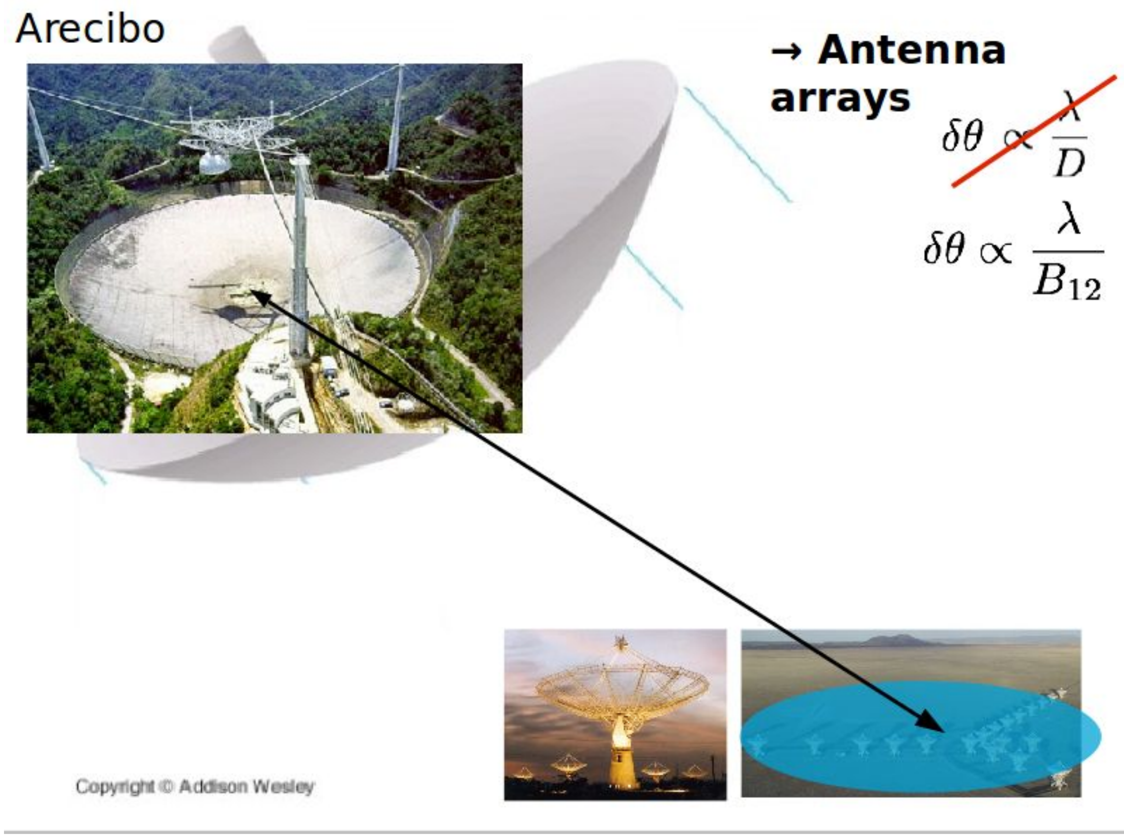
Image credit: Cyril Tasse

3. Why bother with interferometry?



Slide credit:
Julien Girard

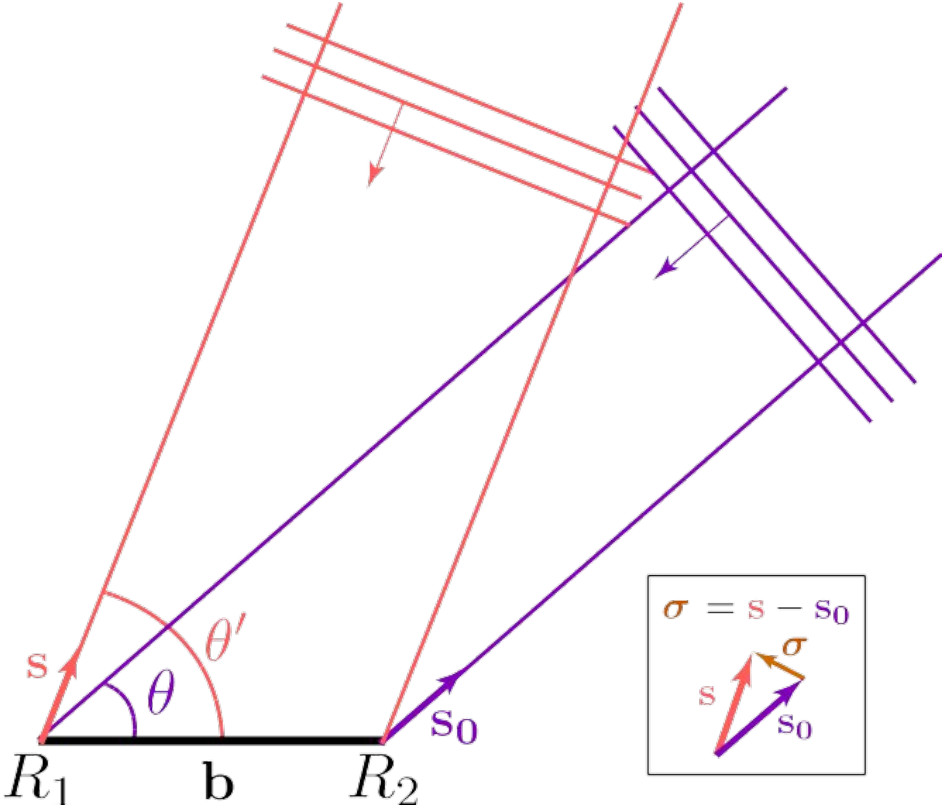
3. Why bother with interferometry?

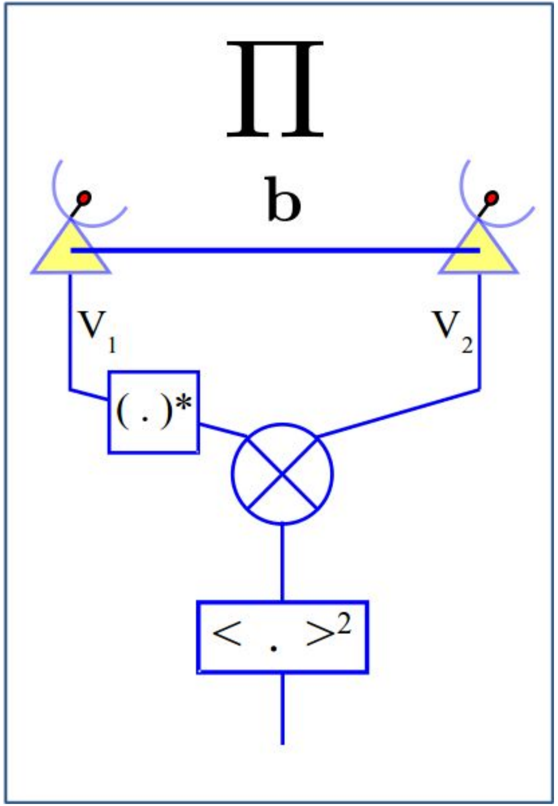


Slide credit:
Julien Girard

4. What is a visibility?

Slide credit:
Julien Girard



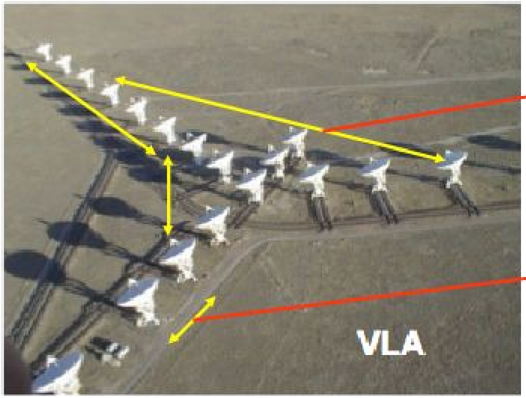


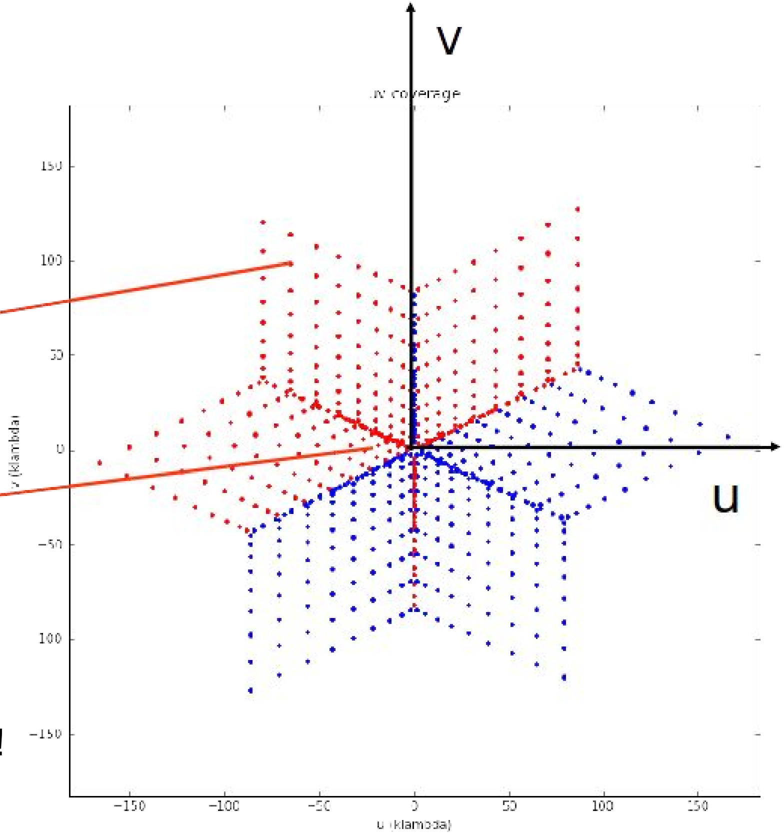
Zernike van Cittert theorem:
Visibility measures ***<u>one Fourier mode</u>***
of the *sky brightness distribution*!

Slide credit:
Julien Girard

5. The UV-plane

Visibilities are Fourier modes:
they live in Fourier space





Radio astronomers refer to it
as *uv*-space: the denser the better!

6. Calibration

Measurements are voltages - not physical flux!

To correct, modern approach is Radio Interferometer's Measurement Equation:

$$\mathbf{V}_{pq} = \mathbf{G}_p \left(\sum_s \mathbf{E}_{sp} \mathbf{K}_{sp} \mathbf{B}_s \mathbf{K}_{sq}^H \mathbf{E}_{sq}^H \right) \mathbf{G}_q^H + \mathbf{N}$$

$$= \sum_s \mathbf{J}_{sp} \mathbf{B}_s \mathbf{J}_{sq}^H + \mathbf{N}$$

(cf. Smirnov 2011 and associated papers)

which implies assuming that measured voltage is linear function of sky signal. All above are 2x2 complex-valued matrices: calibration consists of **solving for $\mathbf{J}_{sp}$**.

6. Calibration

Measurements are voltages - not physical flux!

To correct, modern approach is Radio Interferometer's Measurement Equation:

$$\mathbf{V}_{pq} = \left(\sum_s \mathbf{E}_{sp} \ldots \mathbf{K}^H_{sq} \mathbf{E}^H_{sq} \right) \mathbf{G}^H_q + \mathbf{N}$$

$= \mathbf{G}_p \mathbf{E}_{sp} \mathbf{K}_{sp}$

Noise

$$= \sum_s \mathbf{J}_{sp} \mathbf{B}_s \mathbf{J}^H_{sq} + \mathbf{N}$$

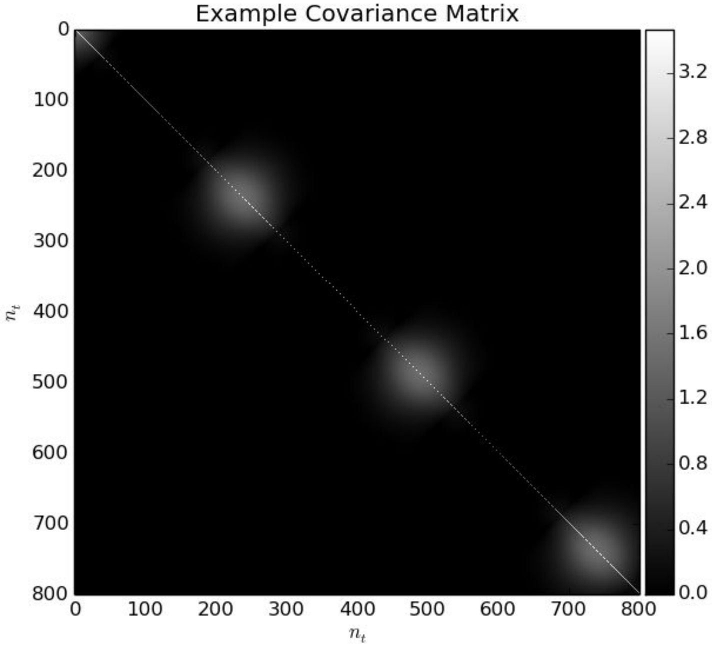
(cf. Smirnov 2011 and associated papers)

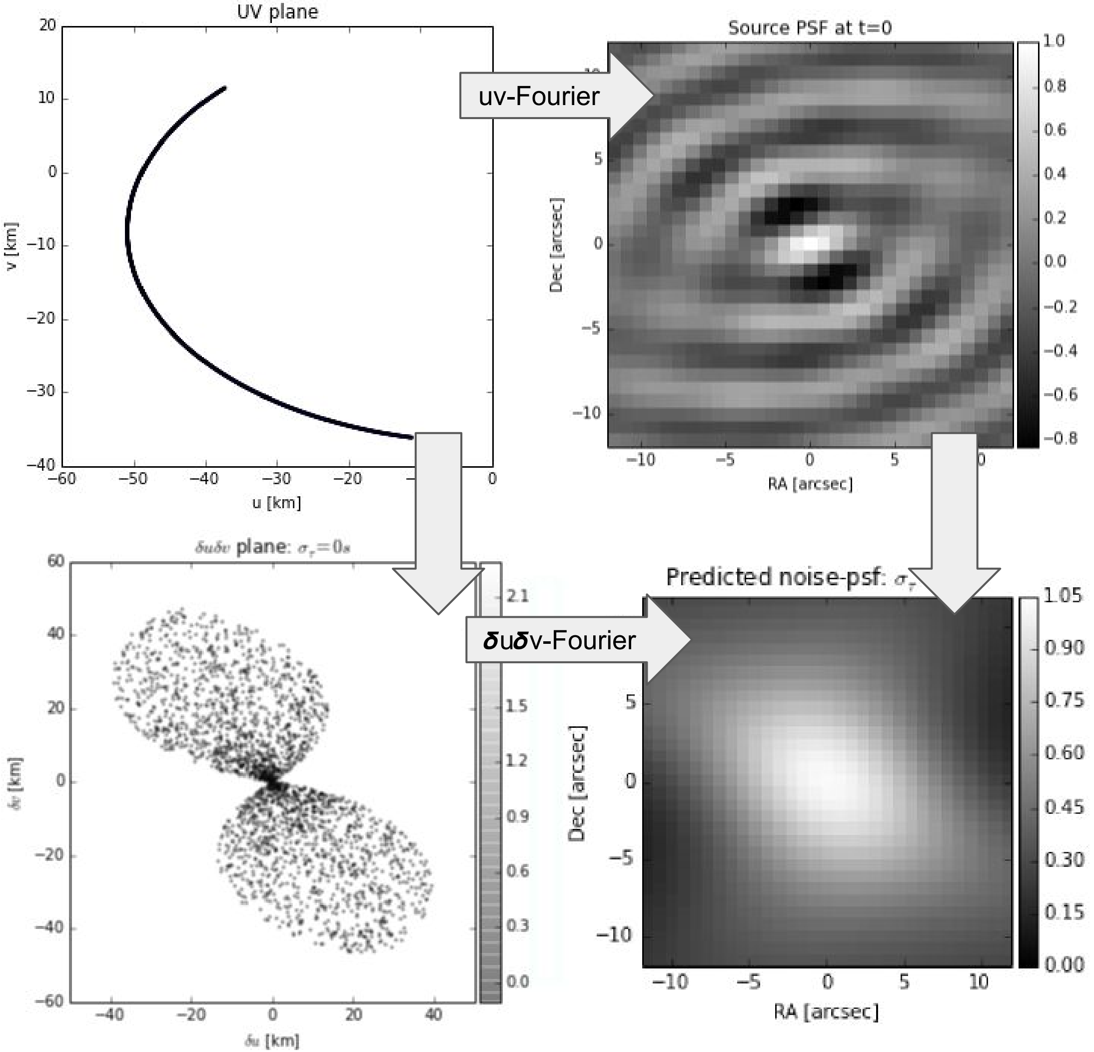
Brightness in direction s

which implies assuming that meas[illegible]d voltage is linear function of sky signal. All above are 2x2 complex-value[illegible]alibration consists of **solving for $\mathbf{J}_{sp}$**.

7. The Noise-PSF

Variance in the image-plane (and covariance between pixels) can be described as the Fourier transform, from ***δ***u***δ***v-space to ***δ***l***δ***m-space, of the visibility covariance matrix.



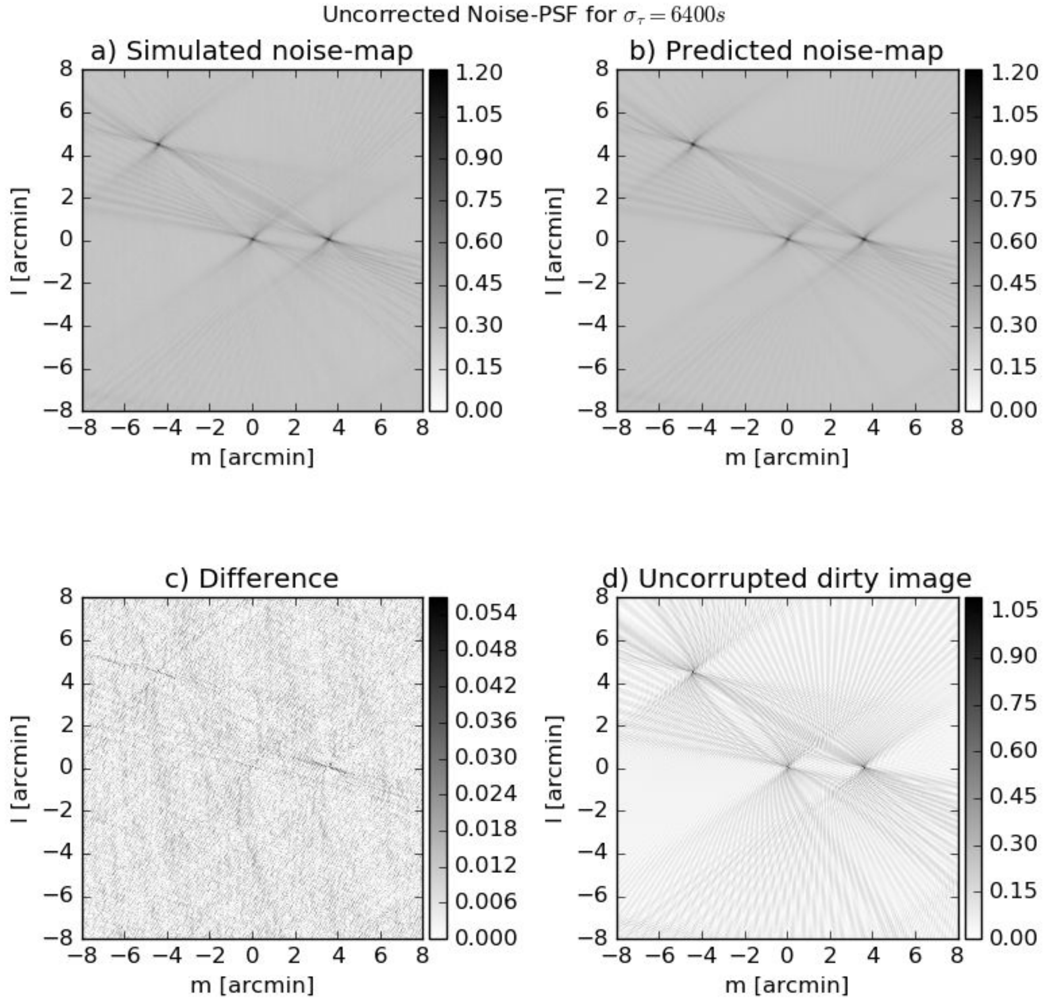


8. The noise-map

i.e. map of the variance in the image-plane. Adequately described by two components:

- Constant noise level, determined by variance in visibilities
- Noise-PSF convolved to all sources in field

if the visibilities consist of spatially incoherent signals added together!



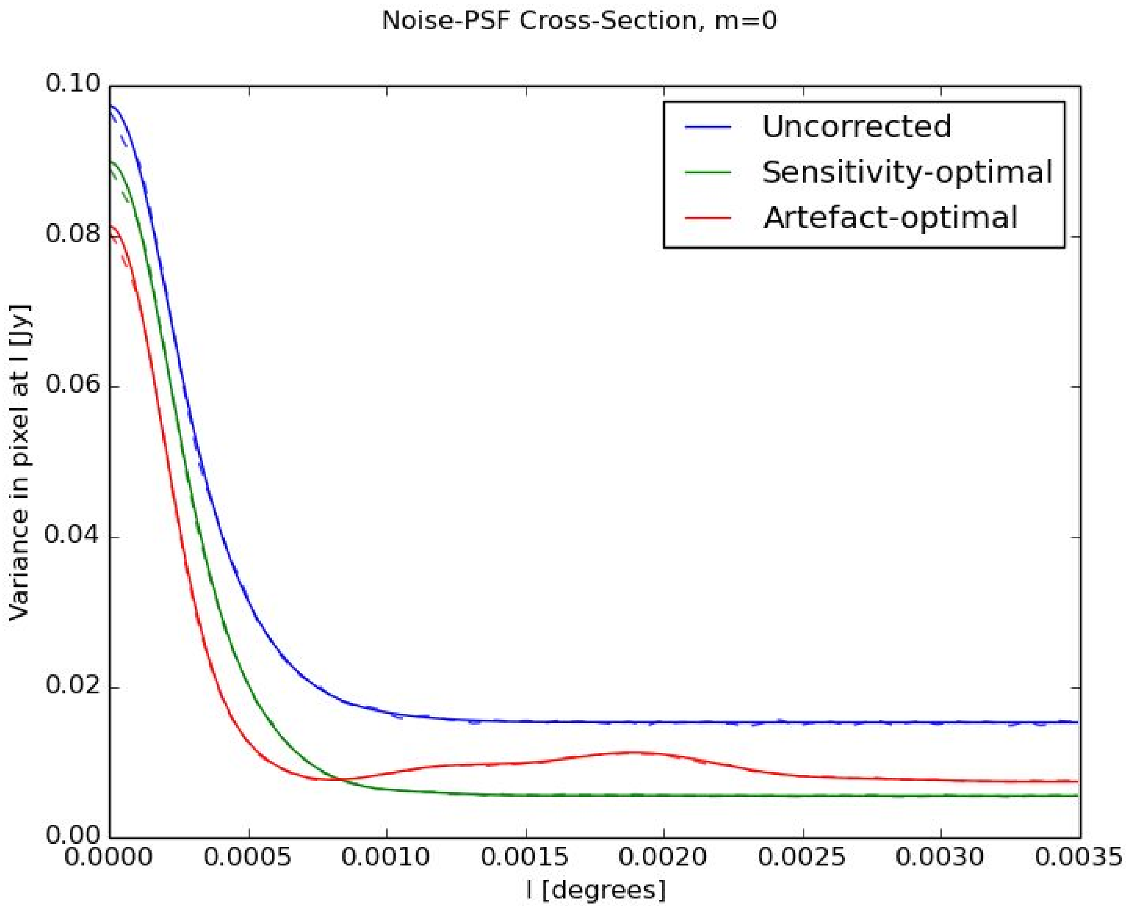
9. Weighting scheme: change the noise-PSF

Sensitivity-optimal:

$$w_b = \frac{1}{\mathrm{Var}\{\tilde{\gamma}_b\}}$$

Artefact-optimal:

$$\boldsymbol{w} = \mathrm{Cov}\{\tilde{\boldsymbol{\gamma}}\}^{-1}\mathbf{1}$$



Right: well-calibrated data

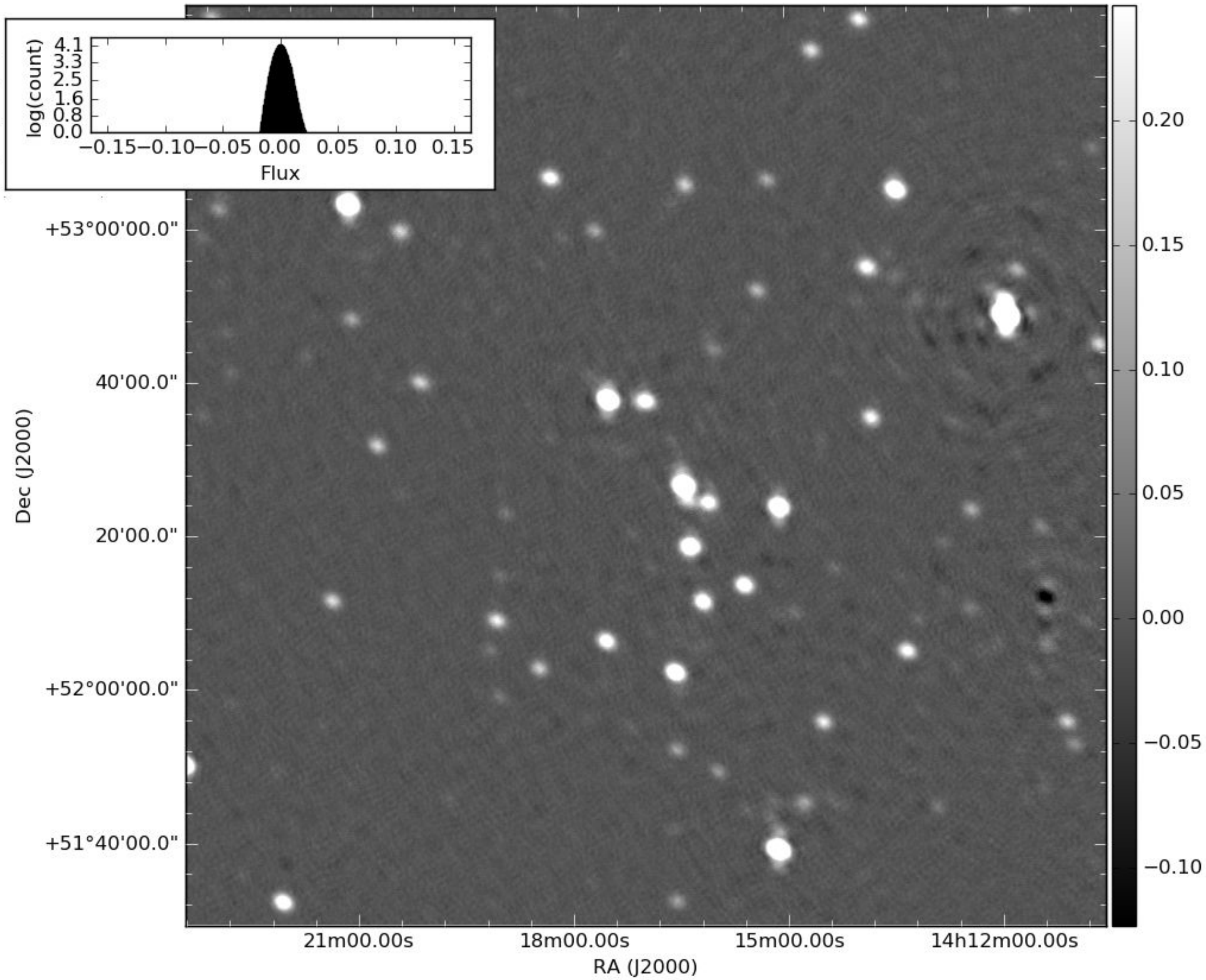
Real data (8-hour LOFAR HBA, 139 MHz, observation of the Bootes deep extragalactic field, 1 data point per 1 second, 8 channels)

Emission:
Synchrotron, free-free

Image:
1.5’’ resolution

Calibration solutions:
1 per 8 seconds per 4 channels

RMS in image:
5.87mJy/beam



Right: poorly-calibrated data

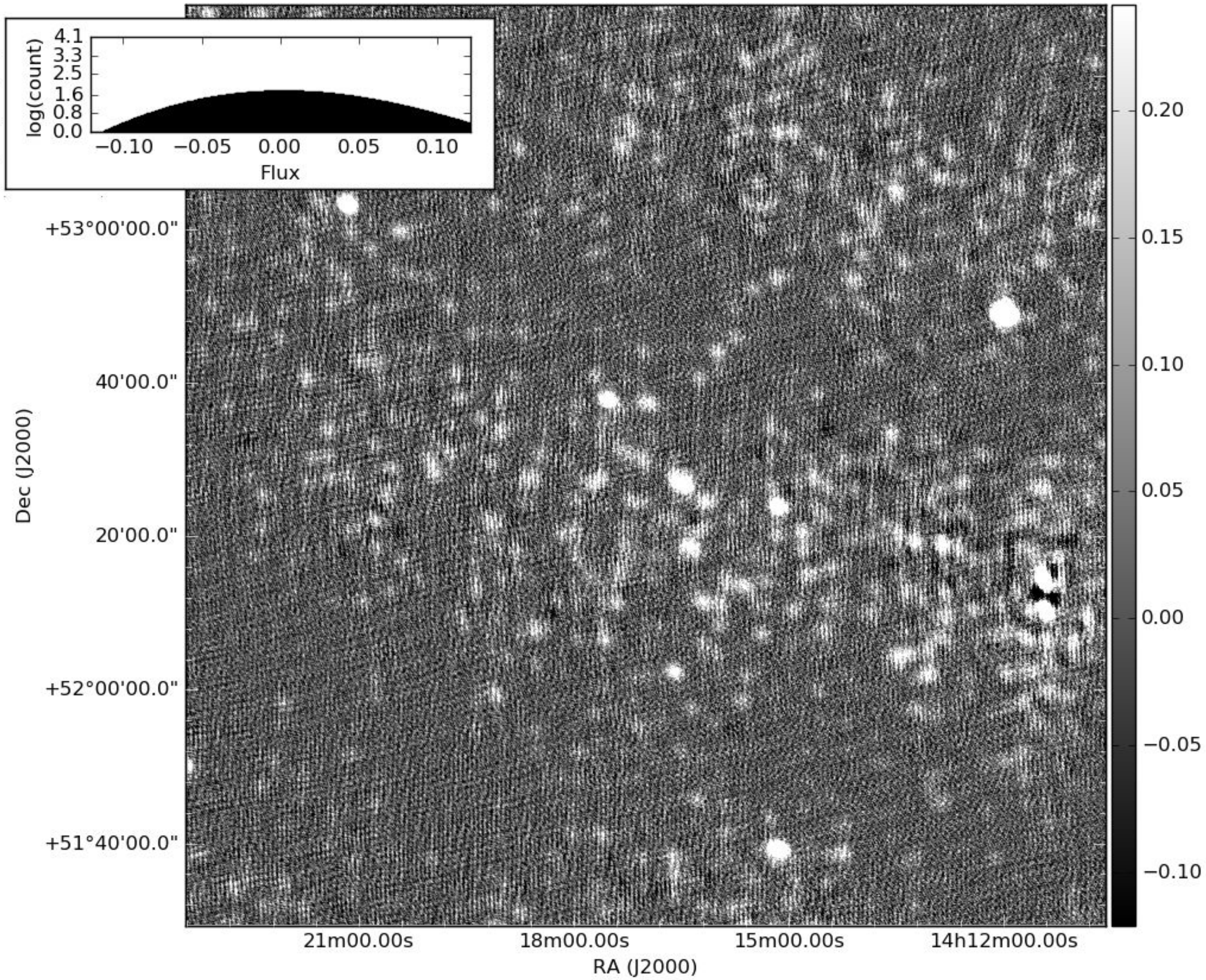
Real data (8-hour LOFAR HBA, 139 MHz, observation of the Bootes deep extragalactic field, 1 data point per 8 second, 8 channels)

Emission:
Synchrotron, free-free

Image:
1.5’’ resolution

Calibration solutions:
1 per 2 minutes per 4 channels

RMS in image:
86.4mJy/beam



Right: sensitivity-optimal

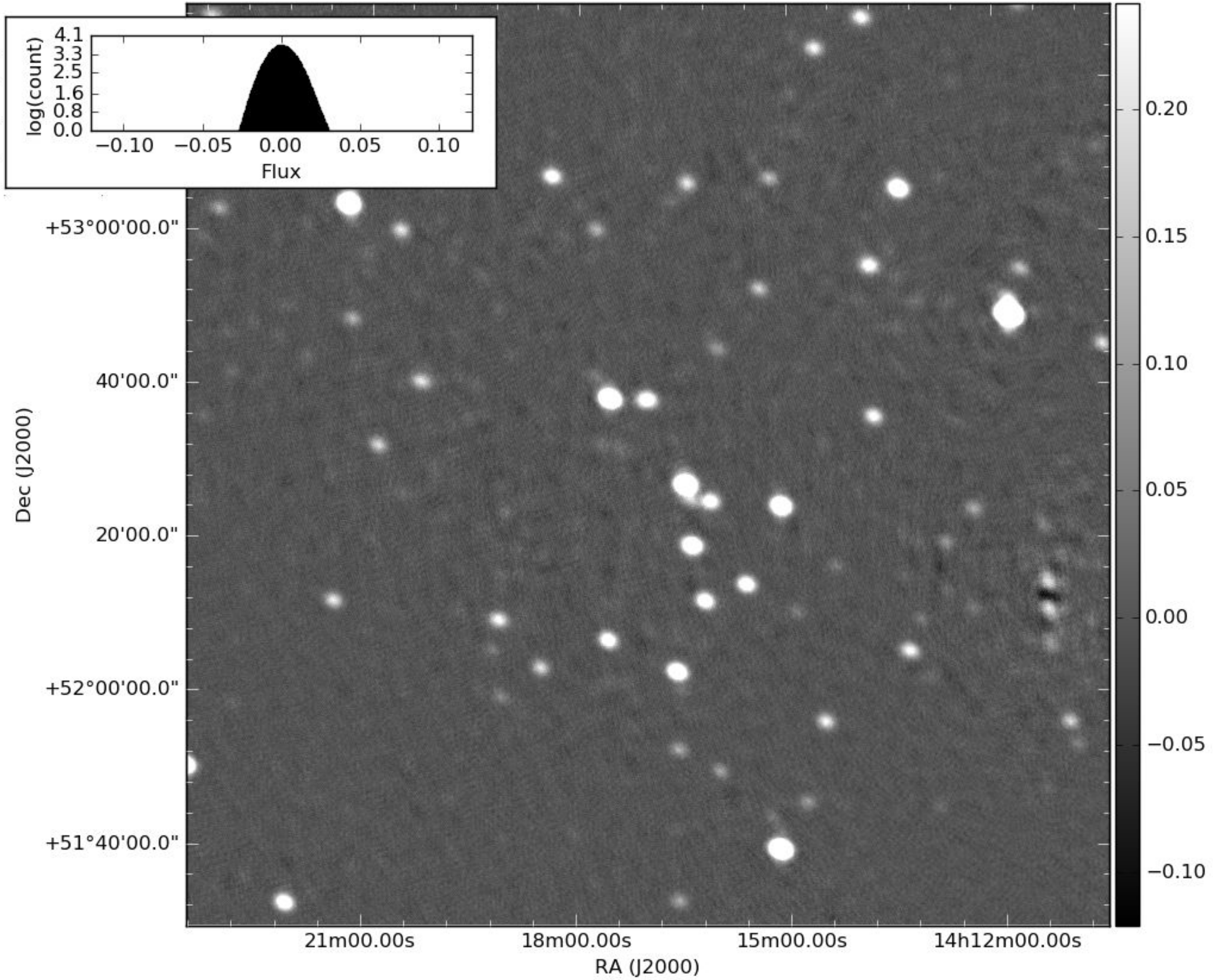
Real data (8-hour LOFAR HBA, 139 MHz, observation of the Bootes deep extragalactic field, 1 data point per 8 second, 8 channels)

Emission:
Synchrotron, free-free

Image:
1.5’’ resolution

Calibration solutions:
1 per 2 minutes per 4 channels

RMS in image:
9.69mJy/beam



Right: artefact-optimal

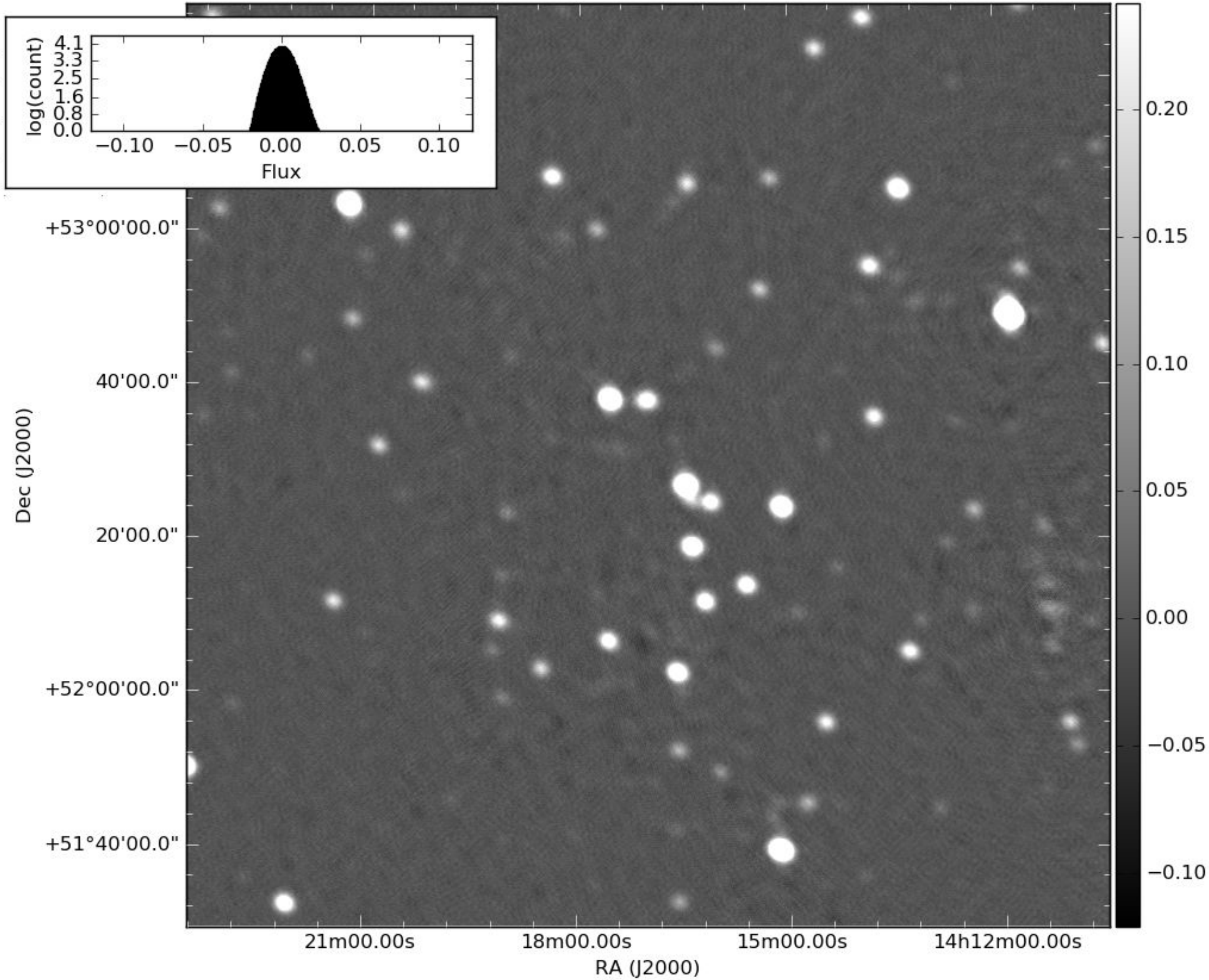
Real data (8-hour LOFAR HBA, 139 MHz, observation of the Bootes deep extragalactic field, 1 data point per 8 second, 8 channels)

Emission:
Synchroton, free-free

Image:
1.5’’ resolution

Calibration solutions:
1 per 2 minutes per 4 channels

RMS in image:
15.8mJy/beam



Right: well-conditioned sens.opt.

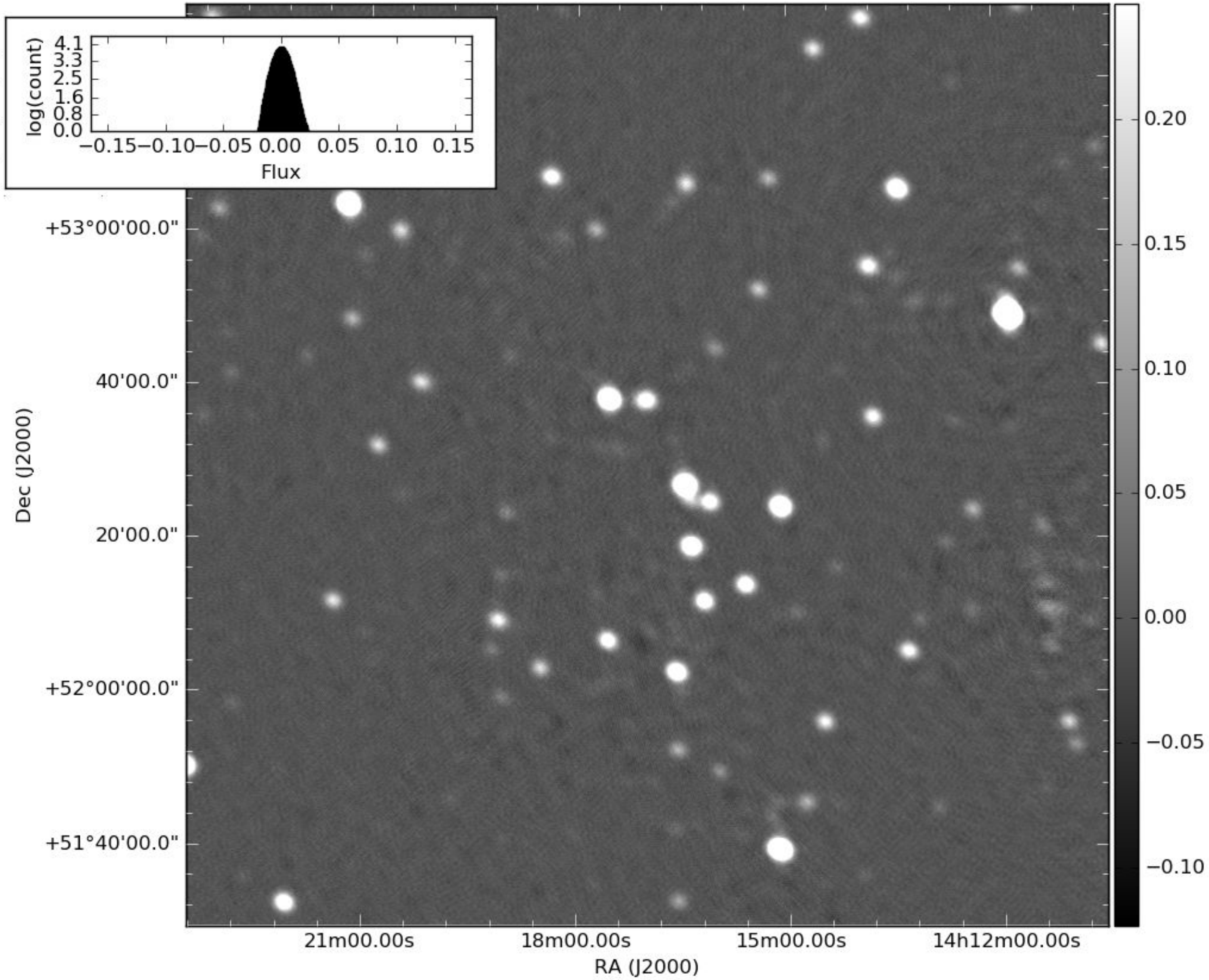
Real data (8-hour LOFAR HBA, 139 MHz, observation of the Bootes deep extragalactic field, 1 data point per 8 second, 8 channels)

Emission:
Synchrotron, free-free

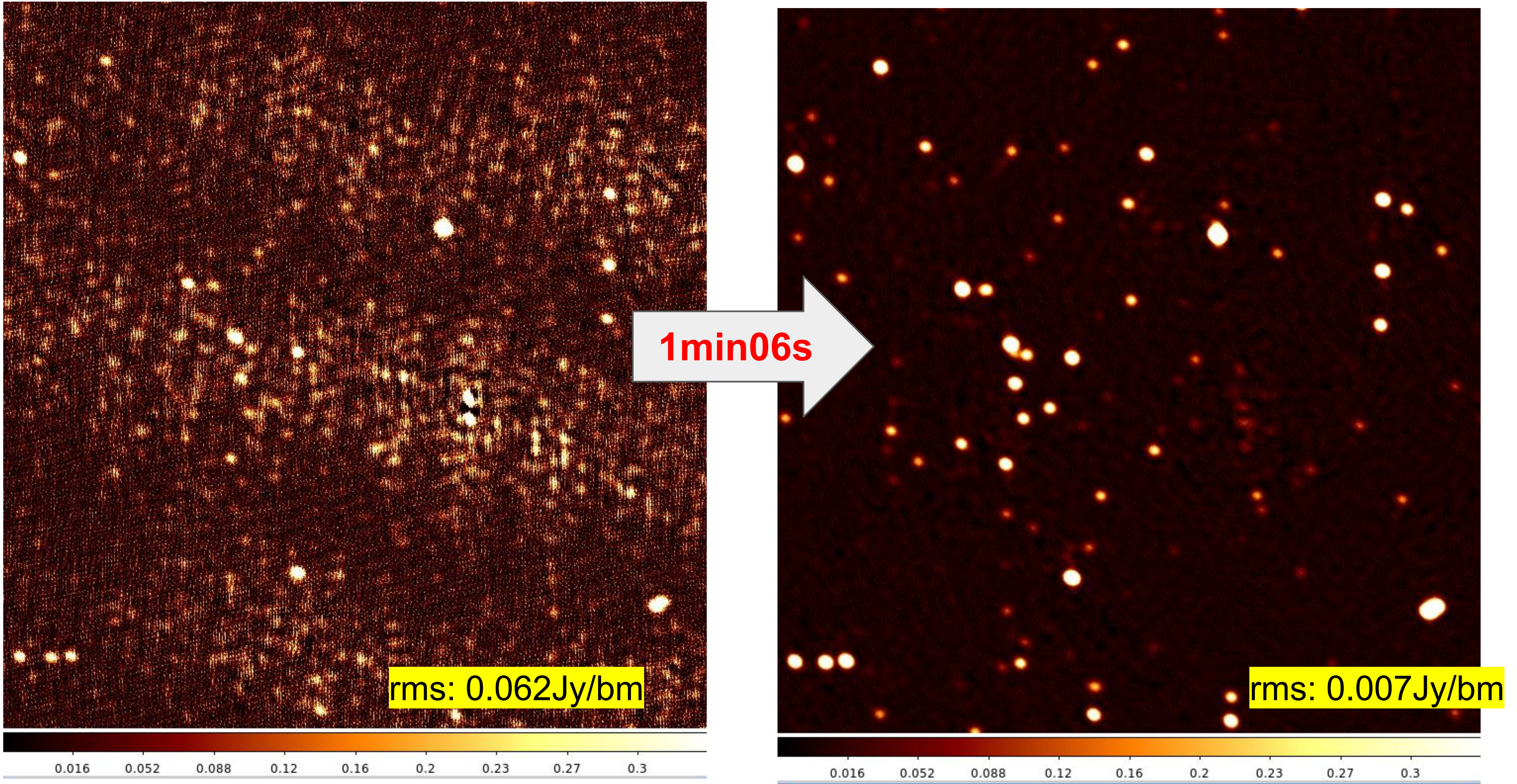
Image:
1.5’’ resolution

Calibration solutions:
1 per 2 minutes per 4 channels

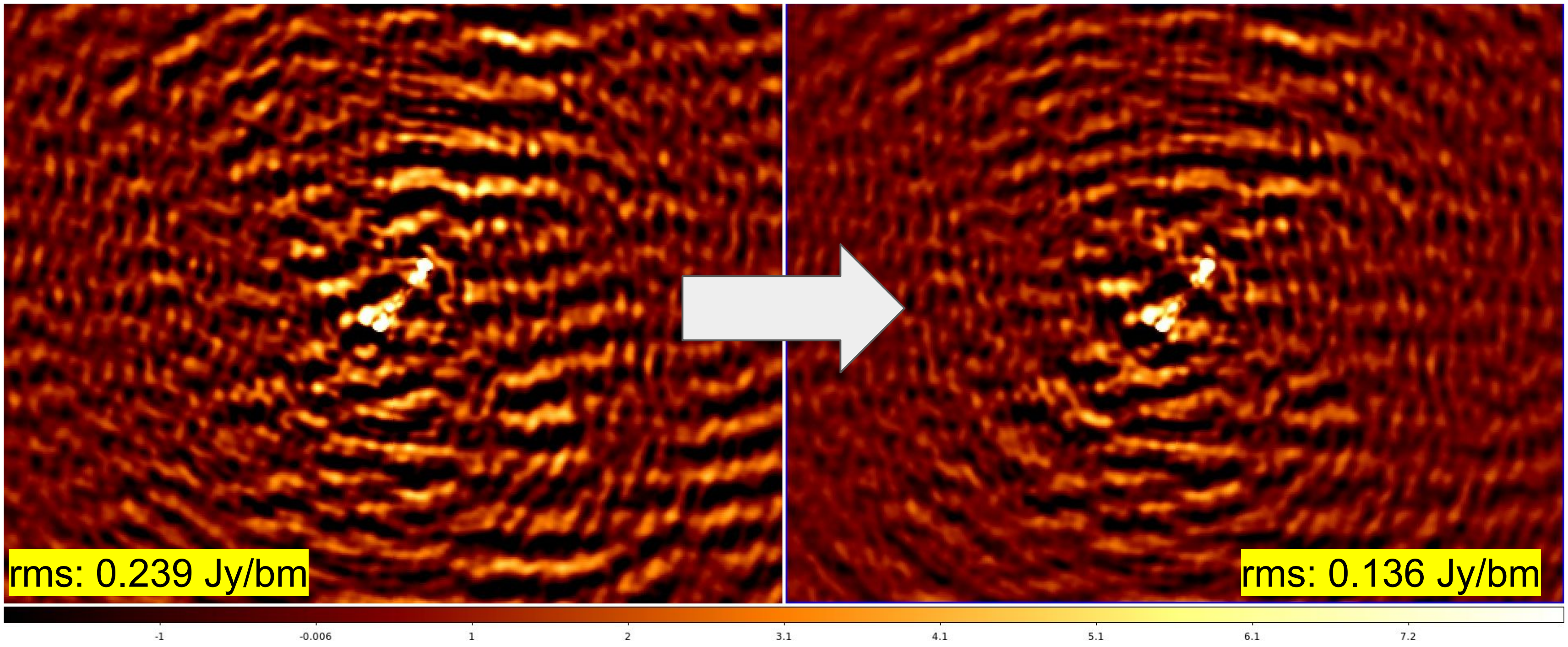
RMS in image:
6.69mJy/beam



11. So it works on badly-calibrated data



12. And on well-calibrated data?



12. And on well-calibrated data?

Factor of 1.8 improvement,

for free

13. Future prospects

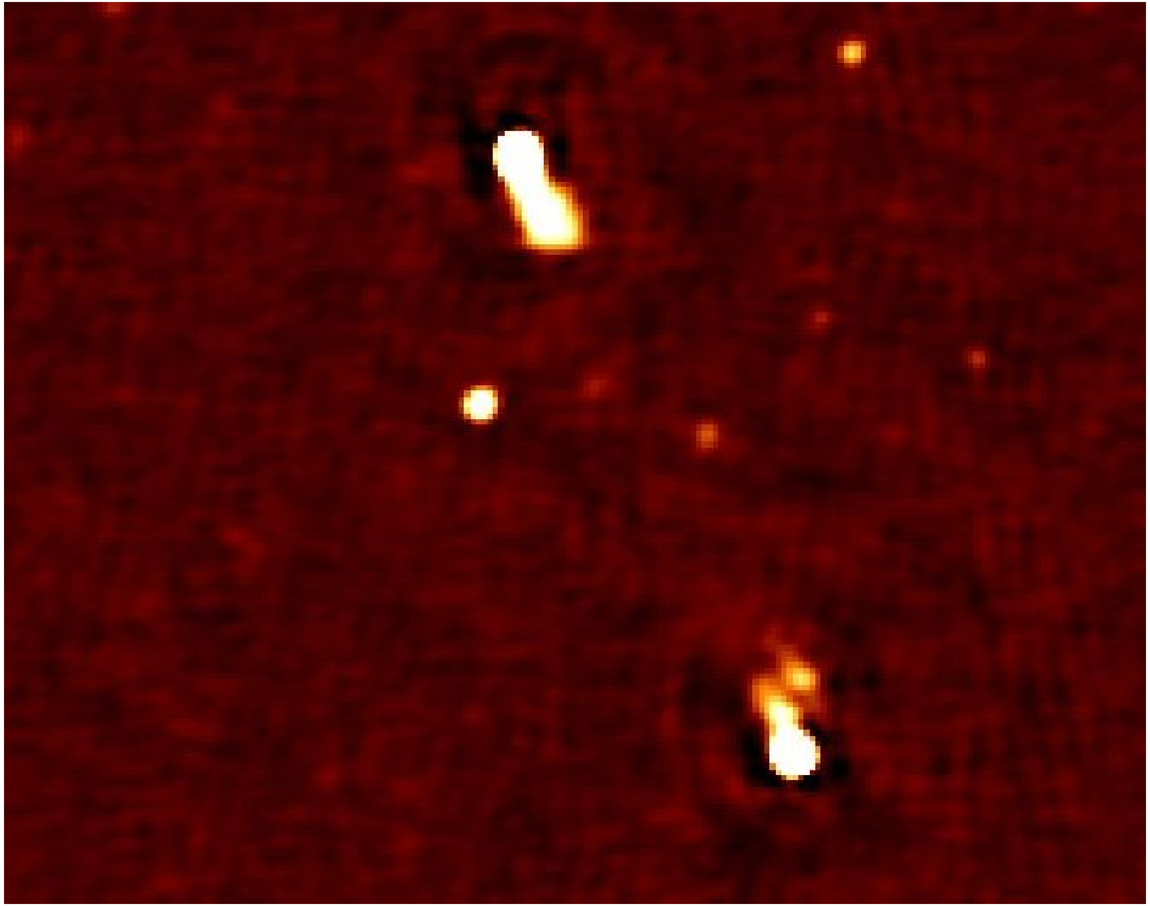
1. Estimating the covariance matrix is tricky: the problem is ill-conditioned. There are ways to improve conditioning which merit further investigation.

2. Impact of sky model incompleteness is still an open question. Relatedly:

3. Understanding the noise-PSF as the *artefact distribution* leads to an obvious question: what is then its relationship to ghosts? This is currently under investigation, in collaboration with Trienko Grobler.

Preprint: https://arxiv.org/abs/1711.00421

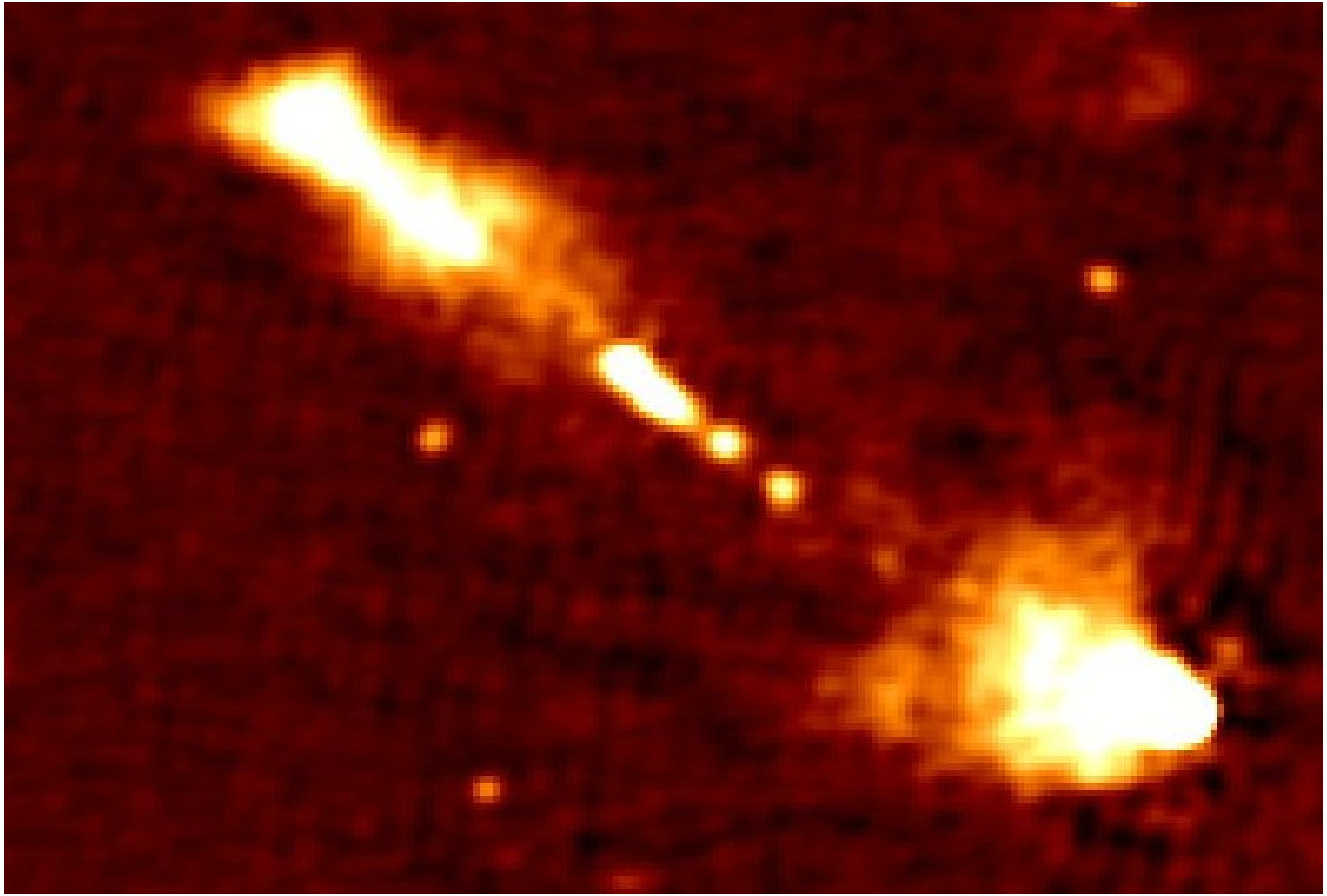
And it lets us see...



Radio galaxy!

Image credit: Cyril Tasse

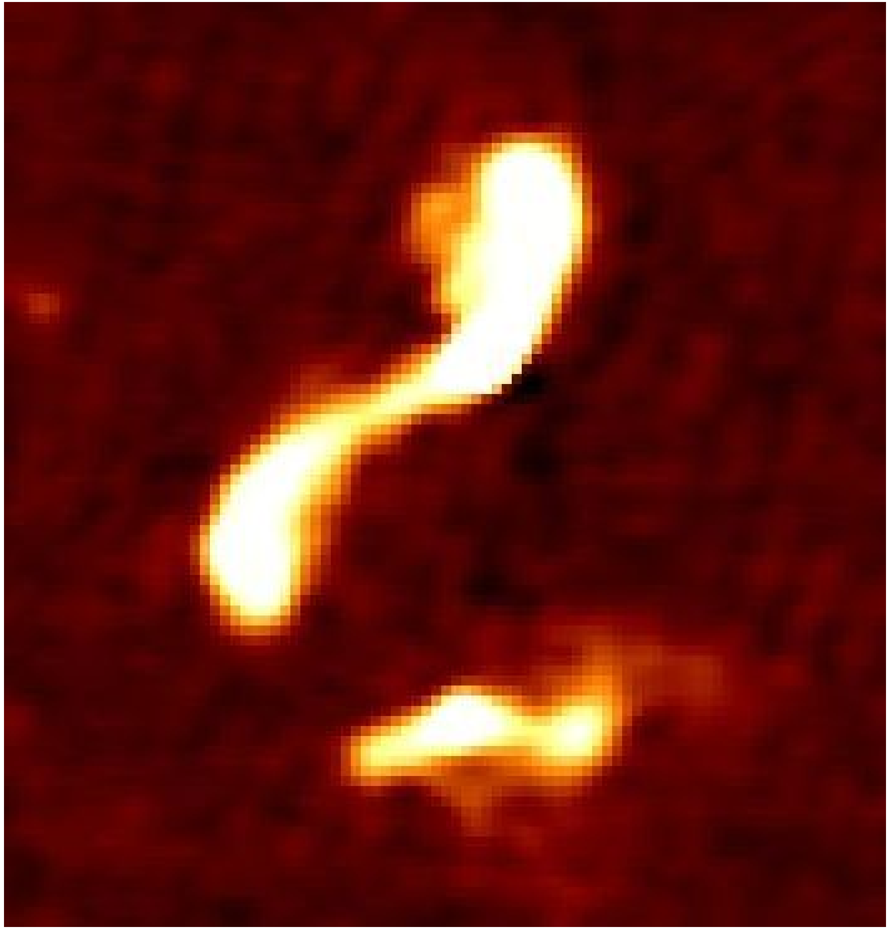
And it lets us see...



Dramatic radio galaxy

Image credit: Cyril Tasse

And it lets us see...

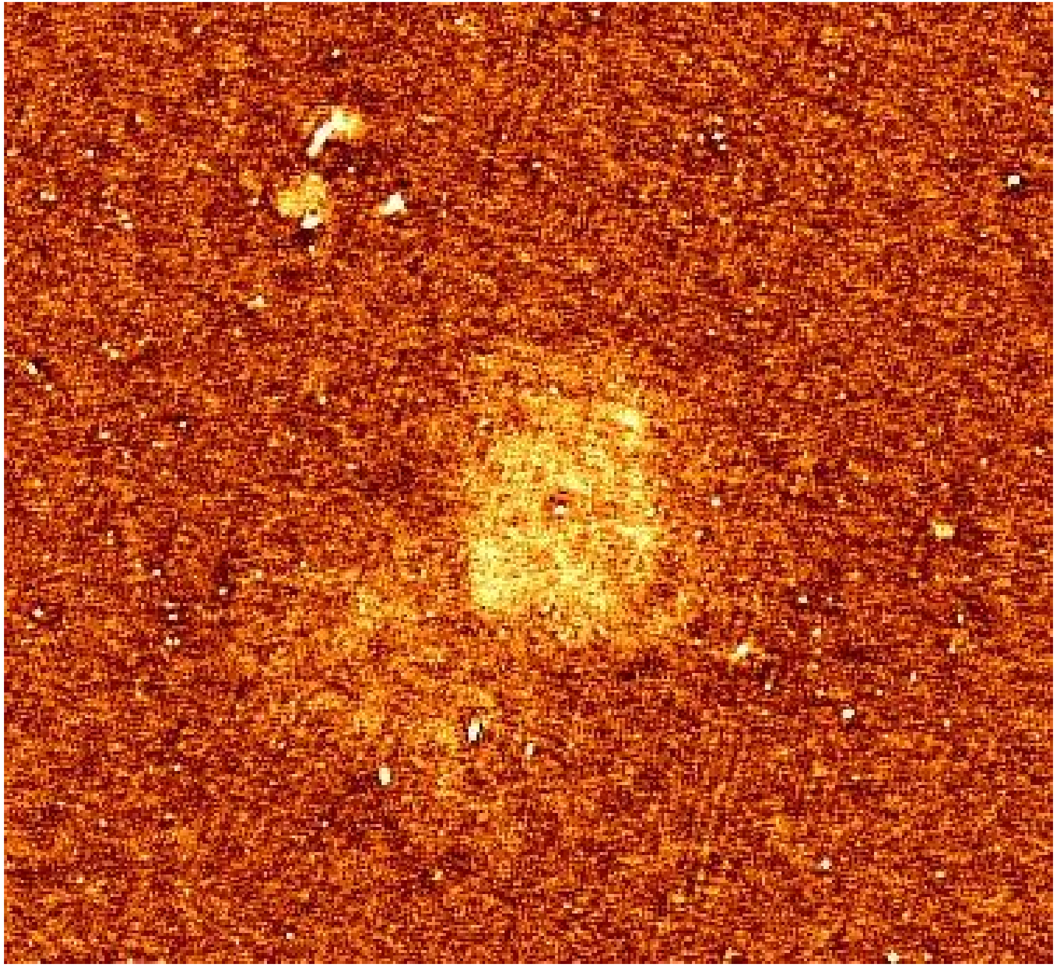


Bent radio galaxy

Bent-tailed (?) radio galaxy

Image credit: Cyril Tasse

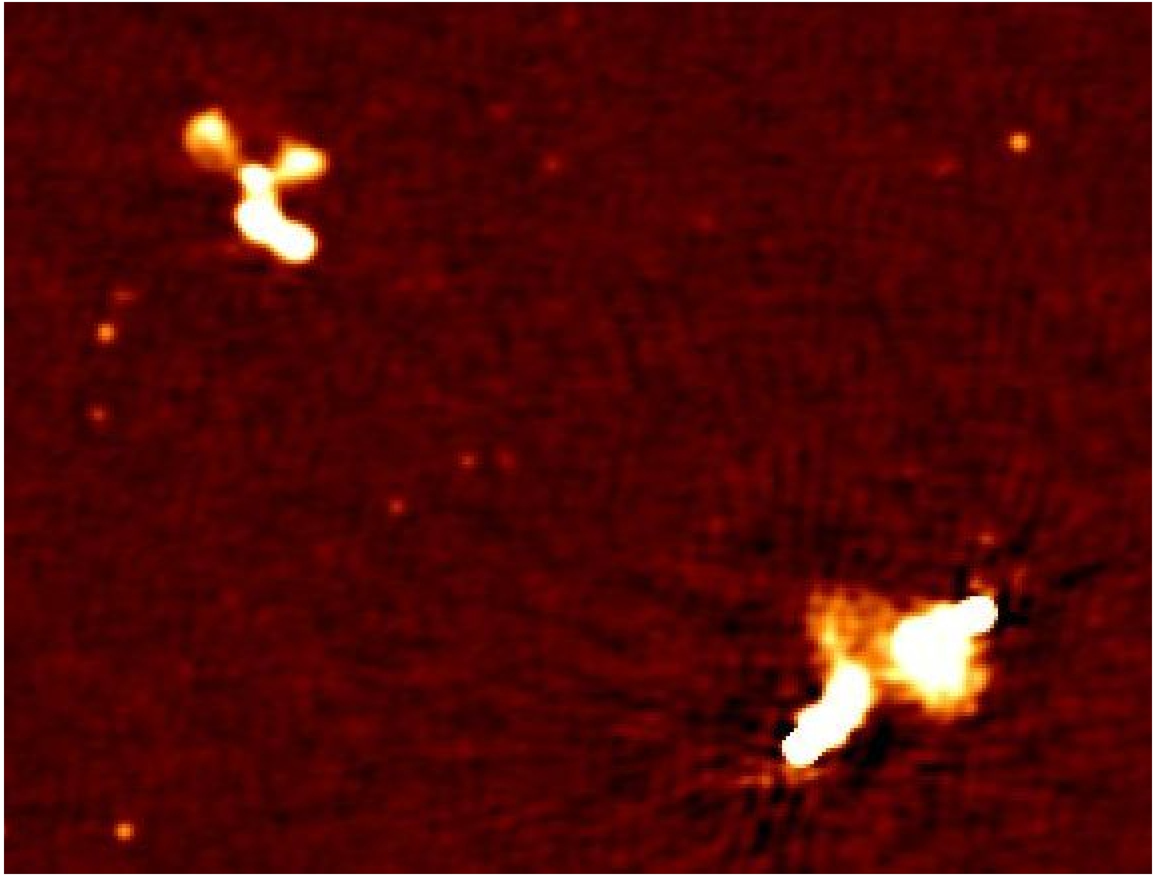
And it lets us see...



Supernova remnant!

Image credit: Cyril Tasse

And it lets us see...



??????????

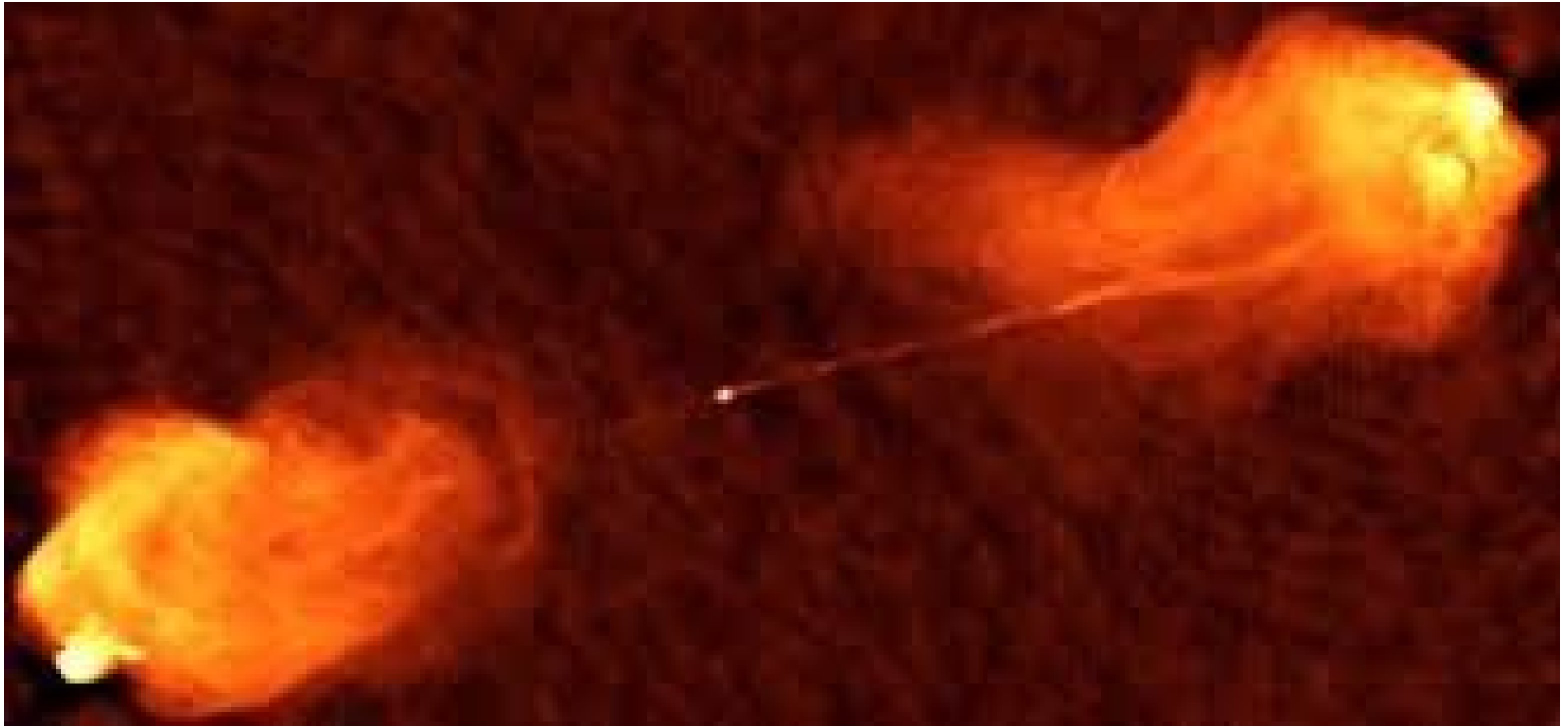
Image credit: Cyril Tasse

Conclusion

Thank you for your time! Questions?

6. What do we measure?

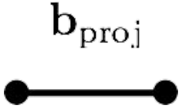
Slide credit:
Julien Girard

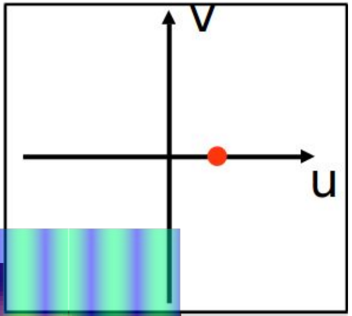


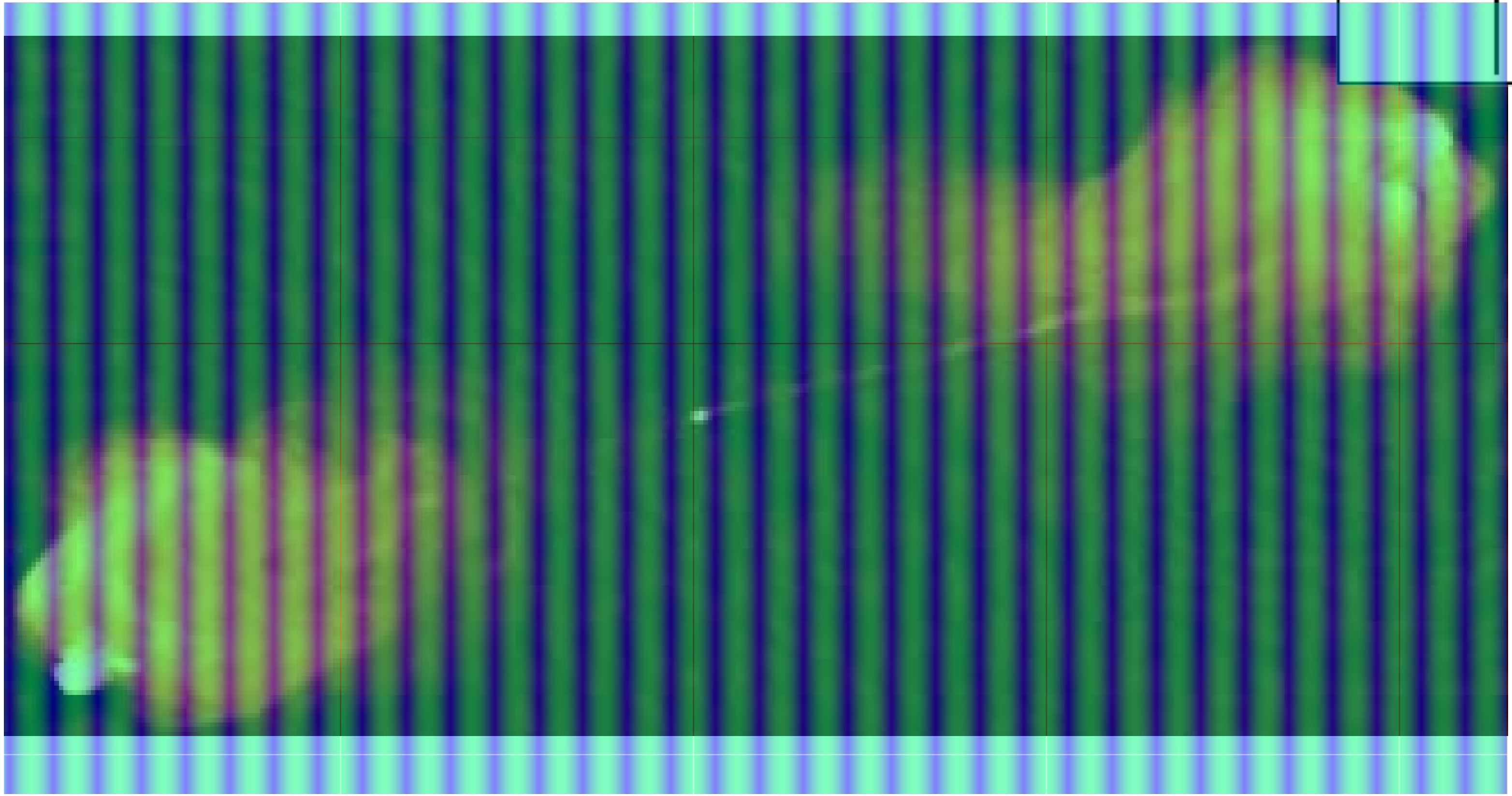
6. What do we measure?

Slide credit:
Julien Girard

UV plane:



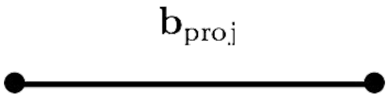


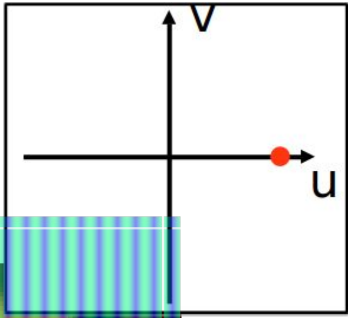


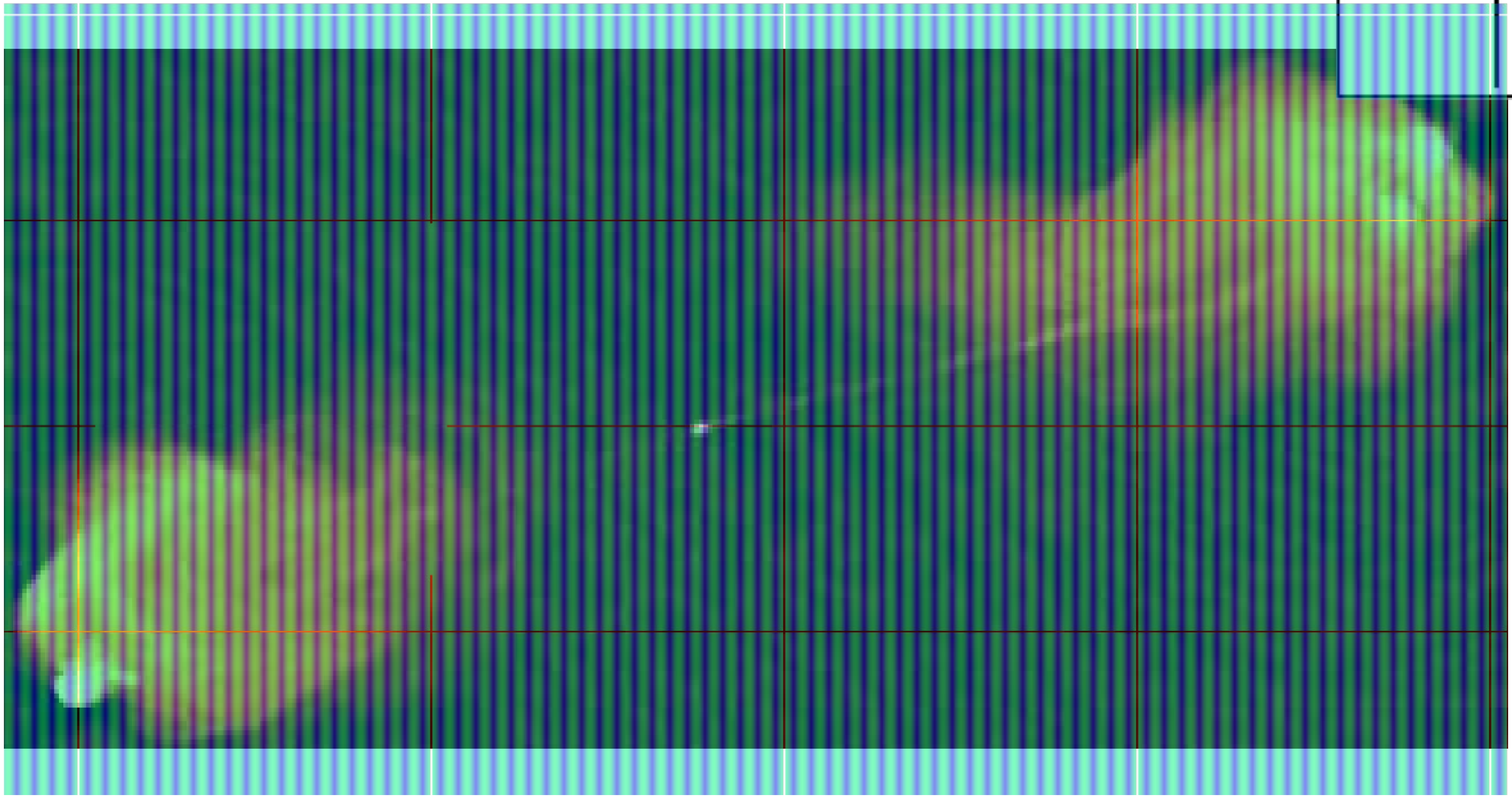
6. What do we measure?

Slide credit:
Julien Girard

UV plane:

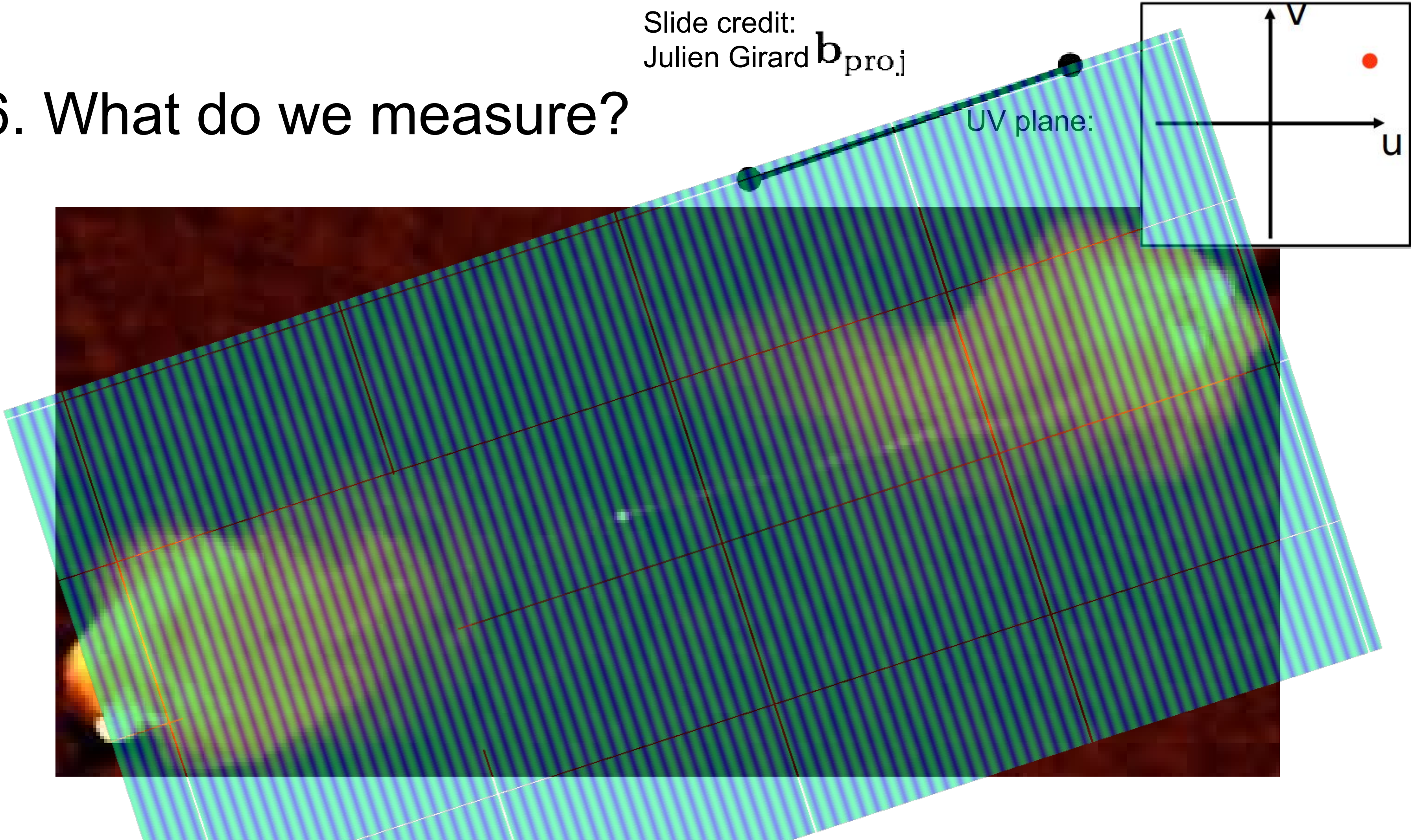






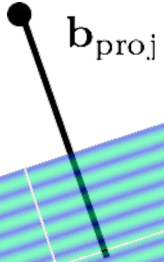
Slide credit:
Julien Girard

6. What do we measure?

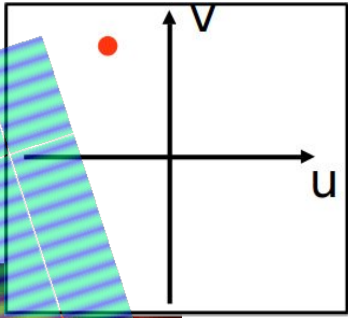


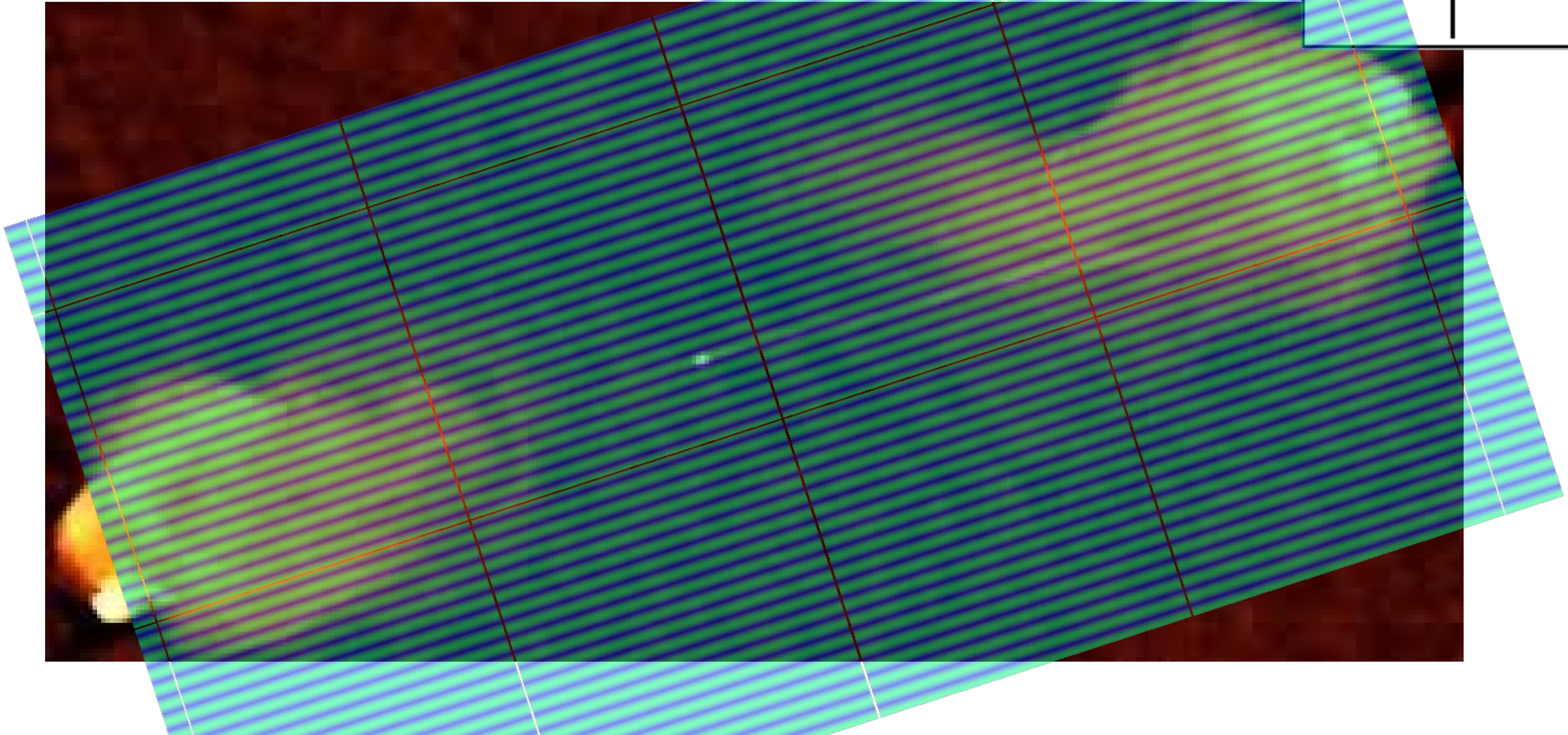
Slide credit:
Julien Girard

6. What do we measure?



UV plane:





7. Calibration

Measurements are voltages - not physical flux!

To correct, modern approach is Radio Interferometer's Measurement Equation:

$$\mathbf{V}_{pq} = \mathbf{G}_p \left(\sum_s \mathbf{E}_{sp} \mathbf{K}_{sp} \mathbf{B}_s \mathbf{K}_{sq}^H \mathbf{E}_{sq}^H \right) \mathbf{G}_q^H + \mathbf{N}$$

$$= \sum_s \mathbf{J}_{sp} \mathbf{B}_s \mathbf{J}_{sq}^H + \mathbf{N}$$

(cf. Smirnov 2011 and associated papers)

which implies assuming that measured voltage is linear function of sky signal. All above are 2x2 complex-valued matrices: calibration consists of **solving for** $\mathbf{J}_{sp}$.

7. Calibration

Fourier sampling with baseline *pq*, direction s

Measurements are voltages - not physica ux!

Noise

To correct, modern approach is Radio Inte erome r's Measuremen quation:

$$\mathbf{V}_{pq} = \mathbf{G}_p \left(\sum_s \mathbf{E}_{sp} \mathbf{K}_{sp} \mathbf{B}_s \mathbf{K}_{sq}^H \mathbf{E}_{sq}^H \right) \mathbf{G}_q^H + \mathbf{N}$$

$$= \sum_s \mathbf{J}_{sp} \mathbf{B}_s \mathbf{J}_{sq}^H + \mathbf{N}$$

(cf. Smirnov 2011 and associated papers)

Brightness in direction *s*

which implies assuming that measured voltage ction of sky signal. All above are 2x2 complex-valued matrices: calib s of **solving for $\mathbf{J}_{sp}$**.

7. Calibration

Measurements are voltages - no [illegible]

Direction-dependent effects acting on antennas p and q

Noise

To correct, modern approach is Radio [illegible]nterferometer's [illegible]easuremen[illegible] [illegible]quation:

$$\mathbf{V}_{pq} = \mathbf{G}_p \left(\sum_s \mathbf{E}_{sp} \mathbf{K}_{sp} \mathbf{B}_s \mathbf{K}_{sq}^H \mathbf{E}_{sq}^H \right) \mathbf{G}_q^H + \mathbf{N}$$

$$= \sum_s \mathbf{J}_{sp} \mathbf{B}_s \mathbf{J}_{sq}^H + \mathbf{N}$$

(cf. Smirnov 2011 and associated papers)

Brightness in direction s

which implies assuming that measured voltage [illegible]ction of sky signal. All above are 2x2 complex-valued matrices: calib[illegible]s of **solving for $\mathbf{J}_{sp}$**.

7. Calibration

Direction-independent effects acting on antennas *p* and *q*

Measurements are voltage - not physical flux!

Noise

To correct, modern approa is Radio Interferometer's Measu emen Equation:

$$\mathbf{V}_{pq} = \mathbf{G}_p \left(\sum_s \mathbf{E}_{sp} \mathbf{K}_{sp} \mathbf{B}_s \mathbf{K}_{sq}^H \mathbf{E}_{sq}^H \right) \mathbf{G}_q^H + \mathbf{N}$$

$$= \sum_s \mathbf{J}_{sp} \mathbf{B}_s \mathbf{J}_{sq}^H + \mathbf{N}$$

(cf. Smirnov 2011 and associated papers)

Brightness in direction *s*

which implies assuming that measured voltage ction of sky signal. All above are 2x2 complex-valued matrices: calib s of **solving for $\mathbf{J}_{sp}$**.

7. Calibration

Measurements are voltages - not physical flux!

To correct, modern approach is Radio Interferometer's Measurement Equation:

$$\mathbf{V}_{pq} = \mathbf{G}_p\left(\sum_s \mathbf{E}_{sp}\mathbf{K}_{sp}\mathbf{B}_s\mathbf{K}_{sq}^H\mathbf{E}_{sq}^H\right)\mathbf{G}_q^H + \mathbf{N}$$

$$= \sum_s \mathbf{J}_{sp}\mathbf{B}_s\mathbf{J}_{sq}^H + \mathbf{N}$$

(cf. Smirnov 2011 and associated papers)

$= \mathbf{G}_p\mathbf{E}_{sp}\mathbf{K}_{sp}$

Noise

Brightness in direction *s*

which implies assuming that meas[illegible]d voltage is linear function of sky signal. All above are 2x2 complex-value[illegible]alibration consists of **solving for $\mathbf{J}_{sp}$**.

8. Noise in interferometric images

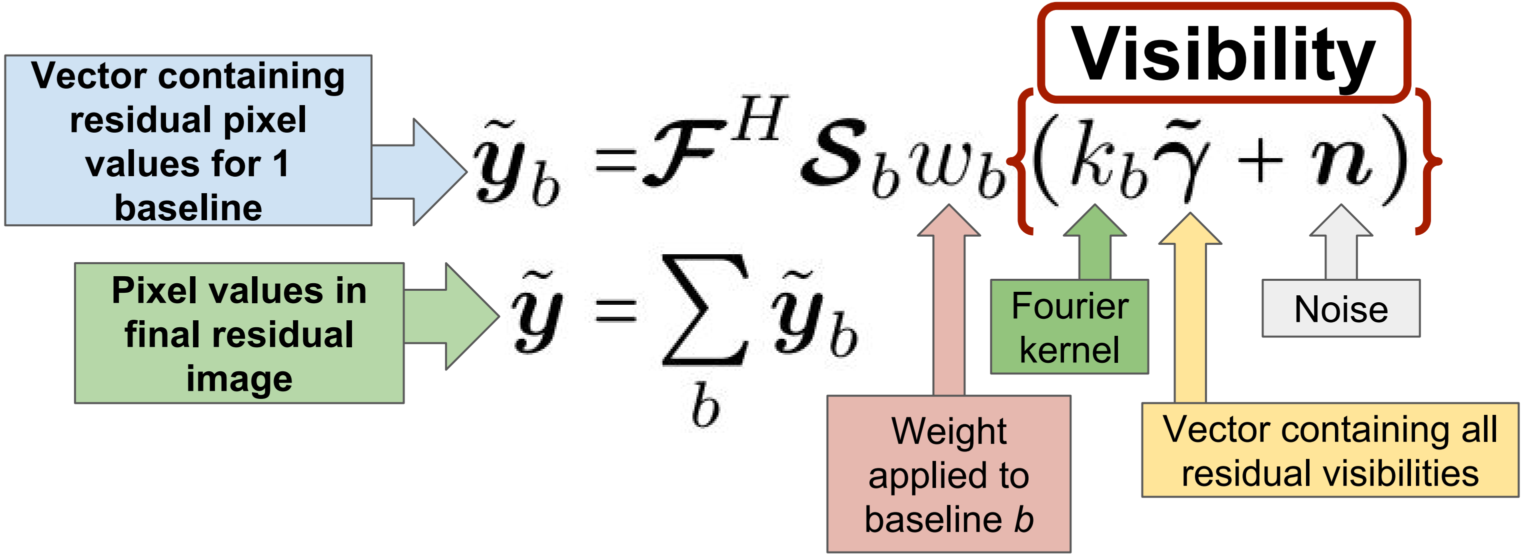
My contribution was to characterise, analytically, how the noise in the image is distributed as a function of the data used to make the image.

$$\tilde{\boldsymbol{y}}_b = \boldsymbol{\mathcal{F}}^H \boldsymbol{\mathcal{S}}_b w_b \left(k_b \tilde{\boldsymbol{\gamma}} + \boldsymbol{n} \right)$$

$$\tilde{\boldsymbol{y}} = \sum_b \tilde{\boldsymbol{y}}_b$$

8. Noise in interferometric images

My contribution was to characterise, analytically, how the noise in the image is distributed as a function of the data used to make the image.



8. Noise in interferometric images

My contribution was to characterise, analytically, how the noise in the image is distributed as a function of the data used to make the image.

Inverse FFT

Gridding matrix, maps a single visibility to the UV-grid

Vector containing residual pixel values for 1 baseline

$$\tilde{\boldsymbol{y}}_b = \underbrace{\boldsymbol{\mathcal{F}}^H \boldsymbol{S}_b}_{\text{Inverse Fourier Transform}} w_b \left(k_b \tilde{\boldsymbol{\gamma}} + \boldsymbol{n}\right)$$

Inverse Fourier Transform

Pixel values in final residual image

$$\tilde{\boldsymbol{y}} = \sum_b \tilde{\boldsymbol{y}}_b$$

9. Noise in interferometric images

Apply Cov{ } operator: gives a relationship between pixel covariance matrix and visibility covariance matrix. This gives us the Cov-Cov relationship:

$$\mathrm{Cov}\{\tilde{\boldsymbol{y}}\} = \sum_d \left(\sum_b \left(\phi^d_{bb}[\mathrm{Cov}\{\tilde{\boldsymbol{\gamma}}\}]_{bb} + w_b^2\sigma^2\right) \boldsymbol{\mathcal{C}}_b + \sum_{b,b' \neq b} \phi^d_{bb'}[\mathrm{Cov}\{\tilde{\boldsymbol{\gamma}}\}]_{bb'} \boldsymbol{\mathcal{F}}_{bb'} \right)$$

9. Noise in interferometric images

Apply Cov{ } operator: gives a relationship between pixel covariance matrix and visibility covariance matrix. This gives us the Cov-Cov relationship:

Variance of visibility b

$$\mathrm{Cov}\{\tilde{\boldsymbol{y}}\} = \sum_d \left(\sum_b \left(\phi^d_{bb} [\mathrm{Cov}\{\tilde{\boldsymbol{\gamma}}\}]_{bb} + w_b^2 \sigma^2\right) \boldsymbol{\mathcal{C}}_b + \sum_{b, b' \neq b} \phi^d_{bb'} [\mathrm{Cov}\{\tilde{\boldsymbol{\gamma}}\}]_{bb'} \boldsymbol{\mathcal{F}}_{bb'} \right)$$

Covariance between visibilities *b* and *b'*

9. Noise in interferometric images

Apply Cov{ } operator: gives a relationship between pixel covariance matrix and visibility covariance matrix. This gives us the Cov-Cov relationship:

Contains squared flux term and weights for visibility *b*

$$\mathrm{Cov}\{\tilde{\boldsymbol{y}}\} = \sum_d \left(\sum_b \left(\phi^d_{bb} [\mathrm{Cov}\{\tilde{\gamma}\}]_{bb} + w_b^2 \sigma^2 \right) \boldsymbol{\mathcal{C}}_b + \sum_{b, b' \neq b} \phi^d_{bb'} [\mathrm{Cov}\{\tilde{\gamma}\}]_{bb'} \boldsymbol{\mathcal{F}}_{bb'} \right)$$

Contains product of model fluxes and weights between visibilities *b* and *b'*

9. Noise in interferometric images

Apply Cov{ } operator: gives a relationship between pixel covar
visibility covariance matrix. This gives us the Cov-Cov relations

Convolution matrix associated with visibility b

$$\mathrm{Cov}\{\tilde{\boldsymbol{y}}\} = \sum_d \left(\sum_b \left(\phi^d_{bb}[\mathrm{Cov}\{\tilde{\boldsymbol{\gamma}}\}]_{bb} + w_b^2\sigma^2\right)\boldsymbol{\mathcal{C}}_b + \sum_{b,b'\neq b} \phi^d_{bb'}[\mathrm{Cov}\{\tilde{\boldsymbol{\gamma}}\}]_{bb'}\boldsymbol{\mathcal{F}}_{bb'} \right)$$

Diagonal is covariance fringe between*b* and *b'*