Application of sparse reconstruction to cosmic magnetism study at low frequencies

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Contents

- Faraday tomography

  - advanced technique for the observation of magnetic fields

- Sparse reconstruction

  - recovery technique for the Faraday spectrum from an ill-conditioned deconvolution problem

- Simulation results

  - test simulation of sparse reconstruction for low frequency coverage

- Summary

Faraday tomography

$$P(\lambda^2) = \int_{-\infty}^{\infty} F(\phi) e^{2i\phi\lambda^2} d\phi$$

polarized emission

(Synchrotron emission)

Faraday rotation

(Brentjens & de Bruyn 2005)

Faraday spectrum : F(ϕ)

F(ϕ) shows the 3D <u>distribution</u> of magnetic fields & polarized sources as a function of Faraday depth

Faraday depth [rad/m^2] $\phi = \int_{there}^{here} n_e(r) B_{||}(r) dr$

Faraday tomography

inverse Fourier transformation

$$\tilde{F} = \frac{1}{\pi}\int_{\lambda^2_{min}}^{\lambda^2_{max}} P(\lambda^2)e^{-2i\phi\lambda^2}d\lambda^2$$

technical problem

・dirty beam

→RM CLEAN - sensitive for Faraday thin source

(Heald et al. 2009, Farnsworth et al. 2011、
Kumazaki et al. 2014, Miyashita et al. 2016)

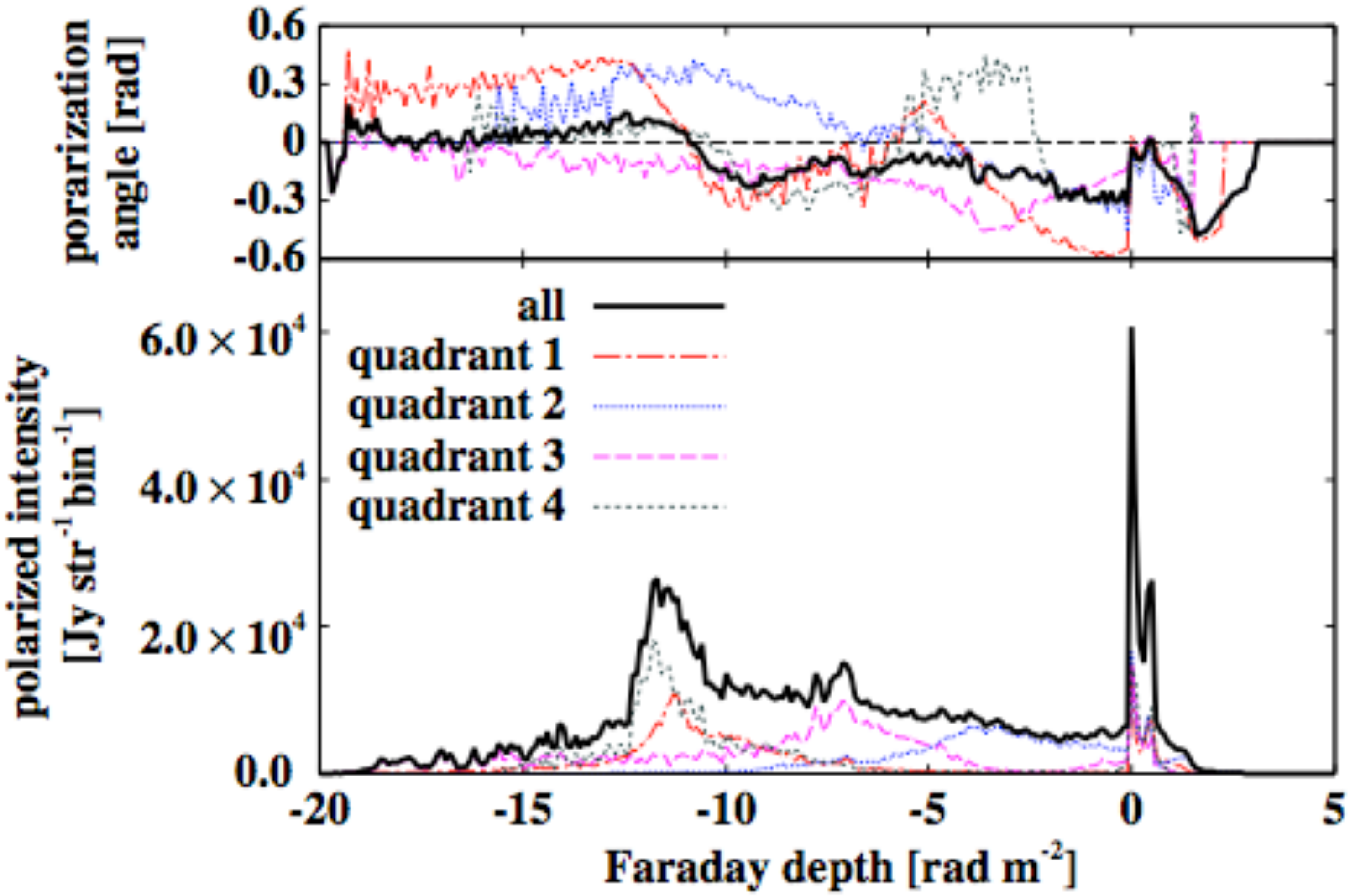
→QU-fit - simple Faraday thin/thick source

(O'Sullivan et al. 2012, Ideguchi et al. 2014, O'Sullivan et al. 2017、
Schnitzeler et al. 2017, Miyashita et al. 2017(submit))

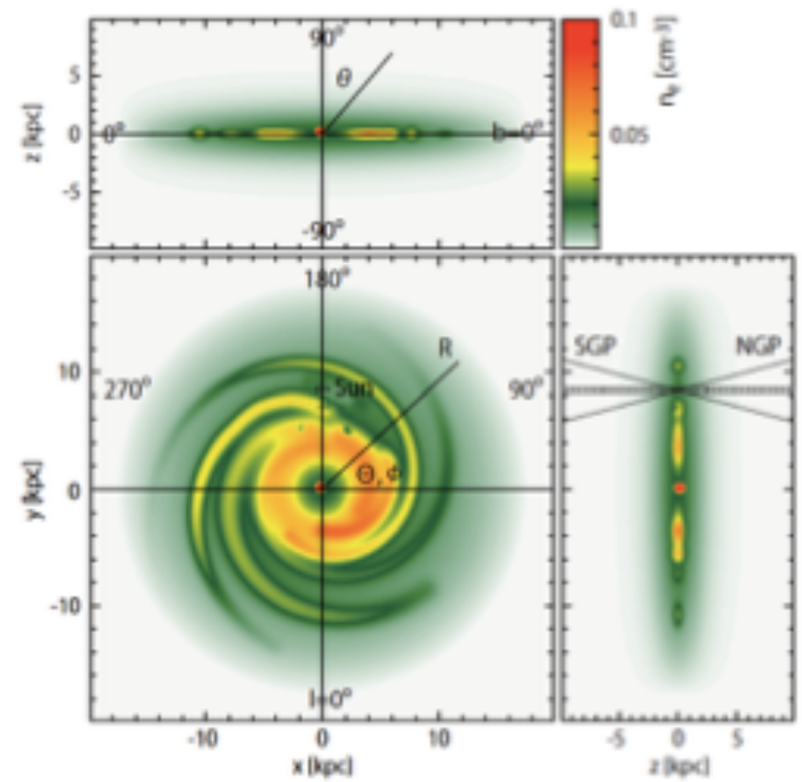
→sparse reconstruction

(Li et al. 2011, Andrecut et al. 2012)

Faraday tomography



(Ideguchi et al. 2014)



spiral galaxy model

(Akahori et al. 2014)

Features

- cosmic/thermal electron density
- global/random magnetic fields
- intrinsic polarization angle

Sparse reconstruction

unknown variable

observable

$$\begin{pmatrix} P(\lambda_1^2) \\ P(\lambda_2^2) \\ P(\lambda_3^2) \\ \vdots \\ P(\lambda_m^2) \end{pmatrix} = \mathbf{A} \times \begin{pmatrix} F(\phi_1) \\ F(\phi_2) \\ F(\phi_3) \\ \vdots \\ \vdots \\ \vdots \\ F(\phi_n) \end{pmatrix}$$

Fourier transform (A)

m < n

Idea

generally, equations can not be solved because of n>m

if <u>most of the F(ϕ) components are 0</u>, ill-posed equations can be solved

sparseness

Sparse solution

how we solve? (Akiyama et al. 2017 in prep)

$$F = min(||\tilde{P} - AF||_2^2 + \Lambda_l ||F||_1 + \Lambda_t ||F||_{tsv})$$

L2 norm — chi-sqared values

L1 norm — sparsity

TSV norm — smoothness

Lp norm : $||F||_p = (\sum_i |F_i|^p)^{\frac{1}{p}}$ $\quad ||F||_{tsv} = \sum_i |F_{i+1} - F_i|^2$

Λ_l, Λ_t : parameters to adjust the level of the regularization

↓

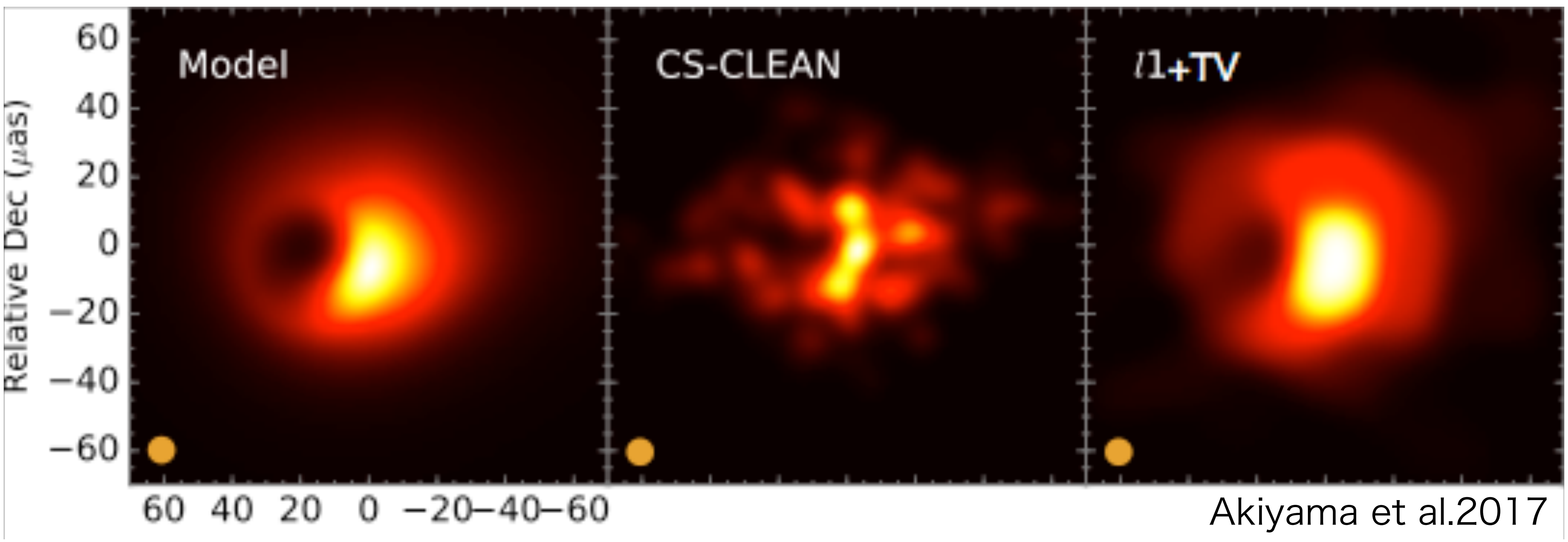
cross validation(CV)

method to fix the appropriate parameters

select the parameter sets of minimum CV chi-squared values

Sparse reconstruction

Applications to M87 simulations
imaging of the BH shadow

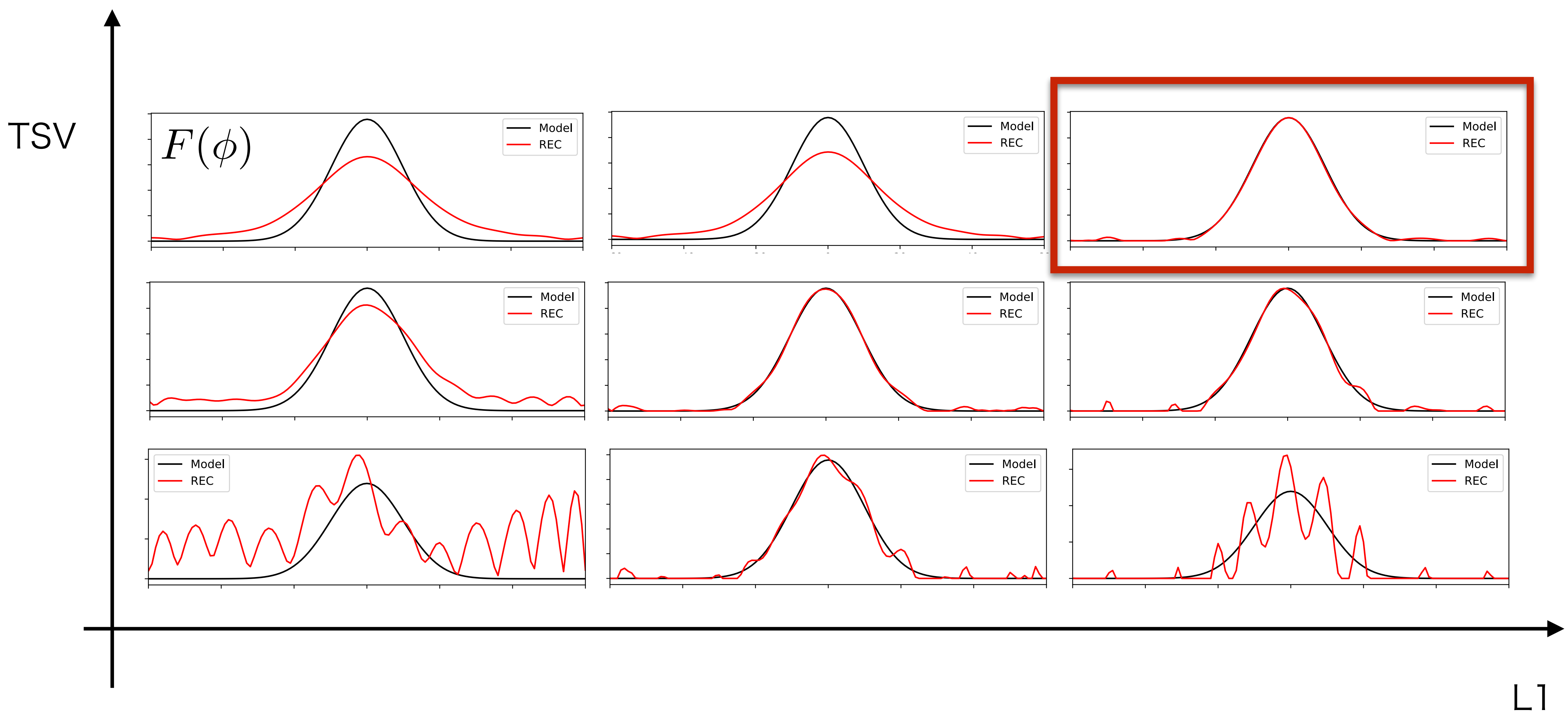


Akiyama et al.2017

BH shadow is reconstructed
using the L1 and TSV regularization

cross validation

1. separate data randomly 10 sub data sets
2. model image with 9/10 of data (training image)
3. calculate chi-squared values of training image and 1/10 of data



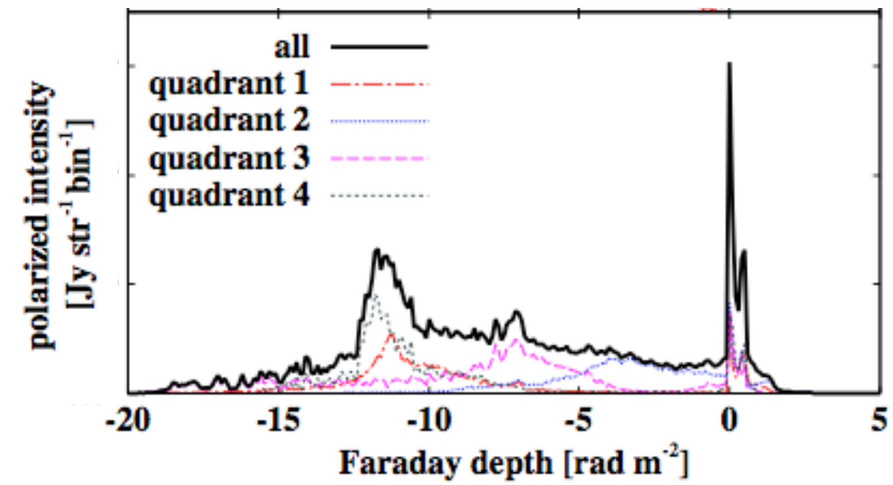
Flowchart simulation

model

$F(\phi)$

calculate galaxy model

$n_e \;\; n_c \;\; B_{||}$



↓

$P(\lambda^2)$

+ noise

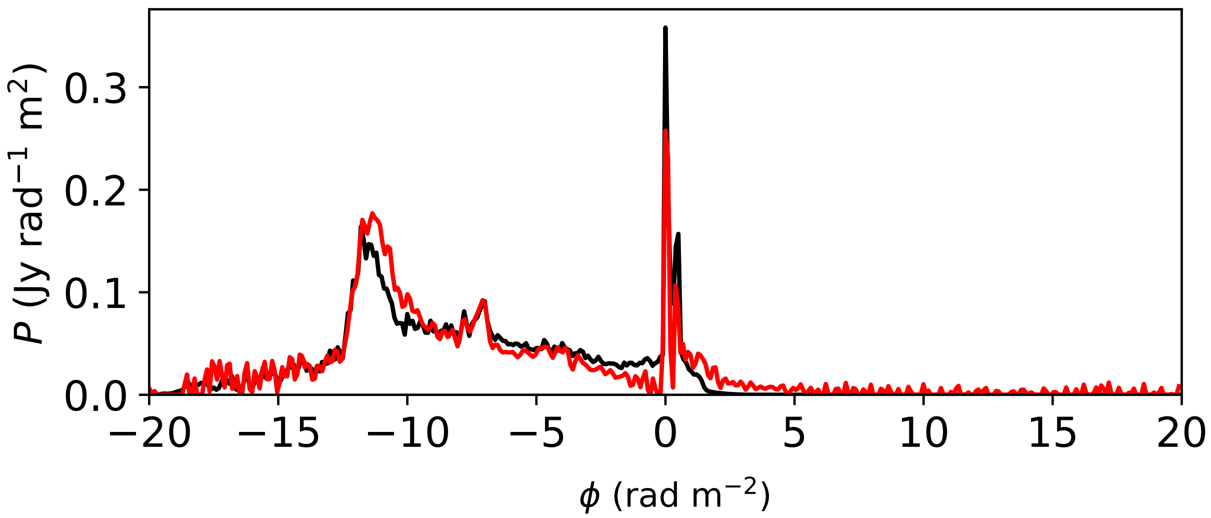
100-300 MHz
700-1800 MHz
100-1800MHz

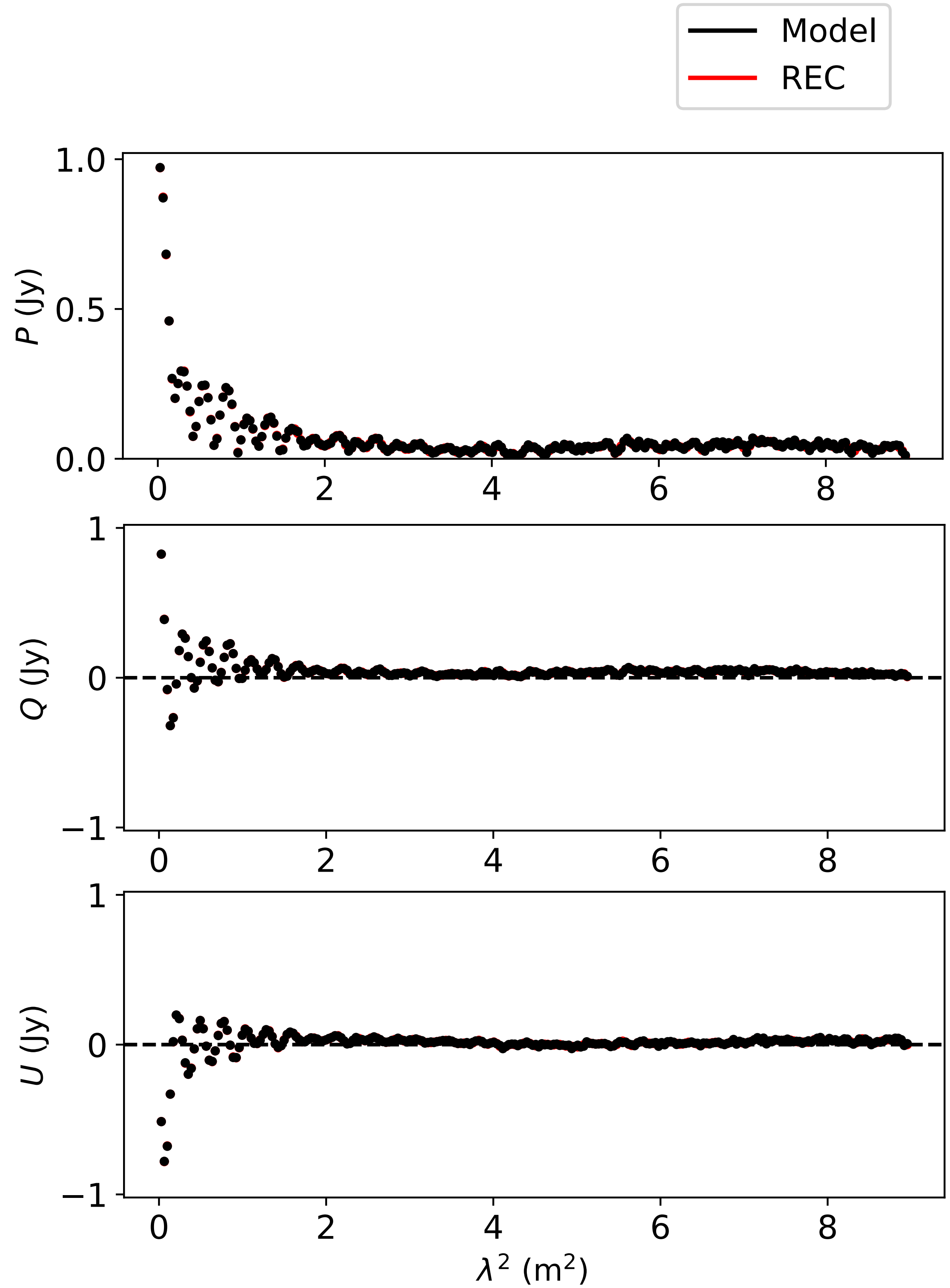
↓ sparse reconstruction

$\tilde{F}(\phi)$

Simulation

100-1800MHz





Details

FWHM : 0.38 [rad/m^2]

Λ_l (sparsity) : 10

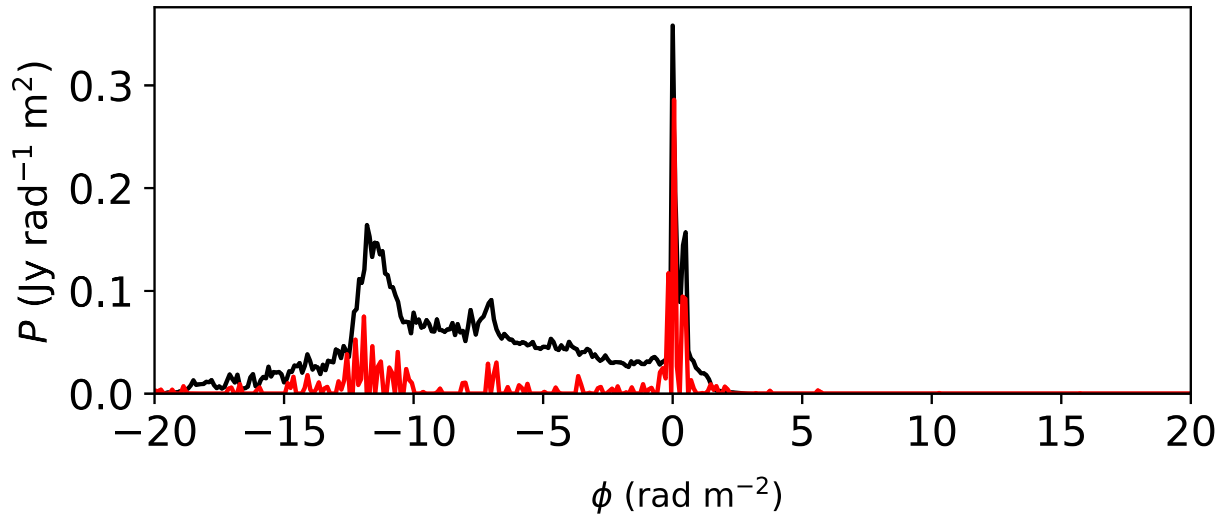
Λ_t (smoothness) : 10

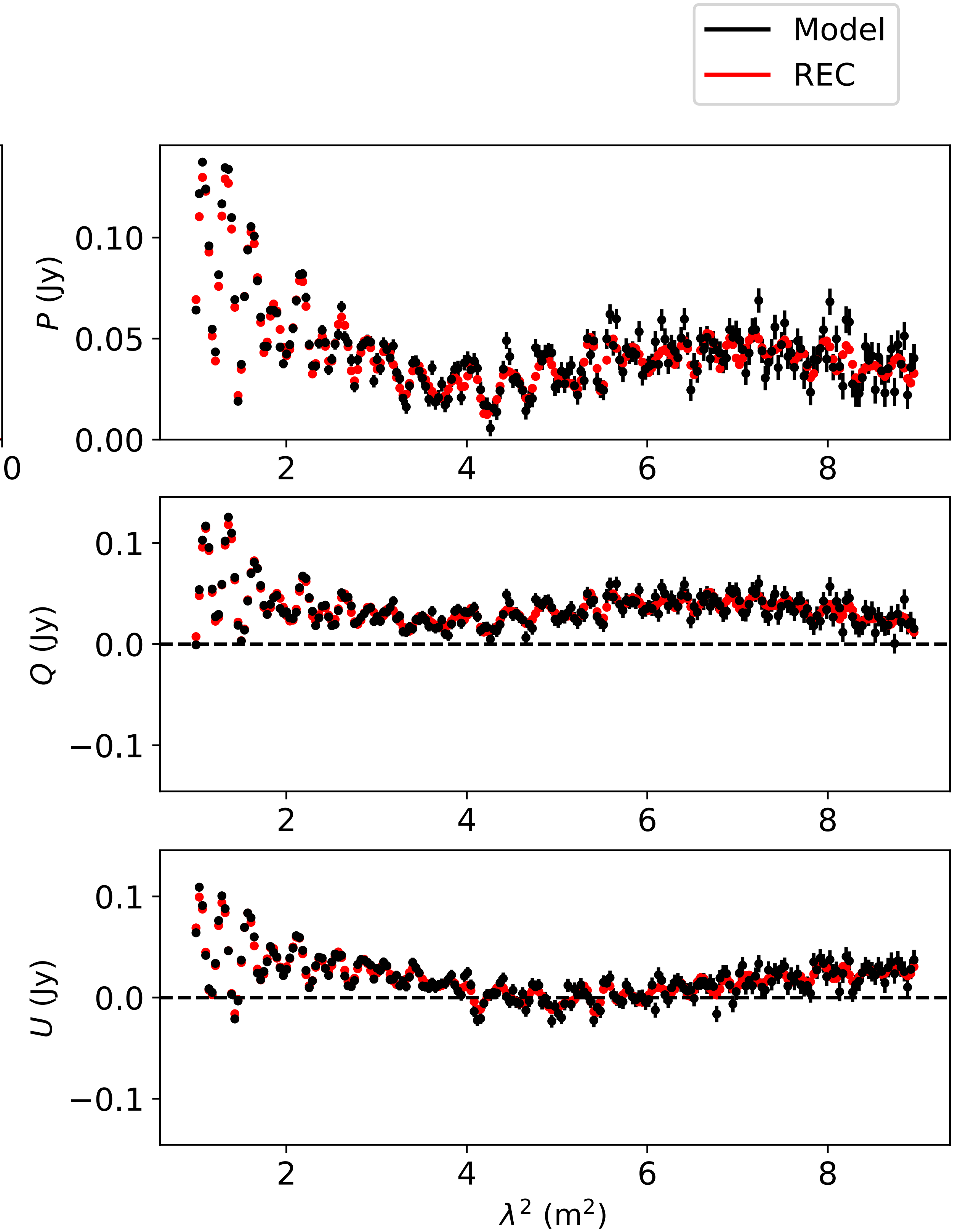
reduced chi-square : 2.17

this coverage can reconstruct almost complete image

Simulation

100-300MHz





Details

FWHM : 0.4 [rad/m^2]

Λ_l (sparsity) : 10

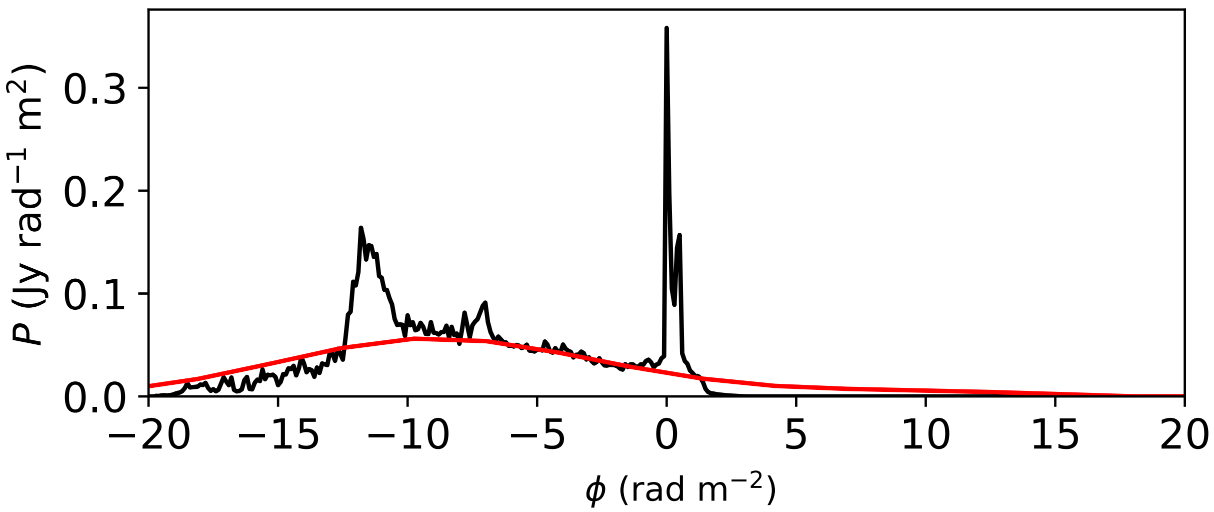
Λ_t (smoothness) : 0.01

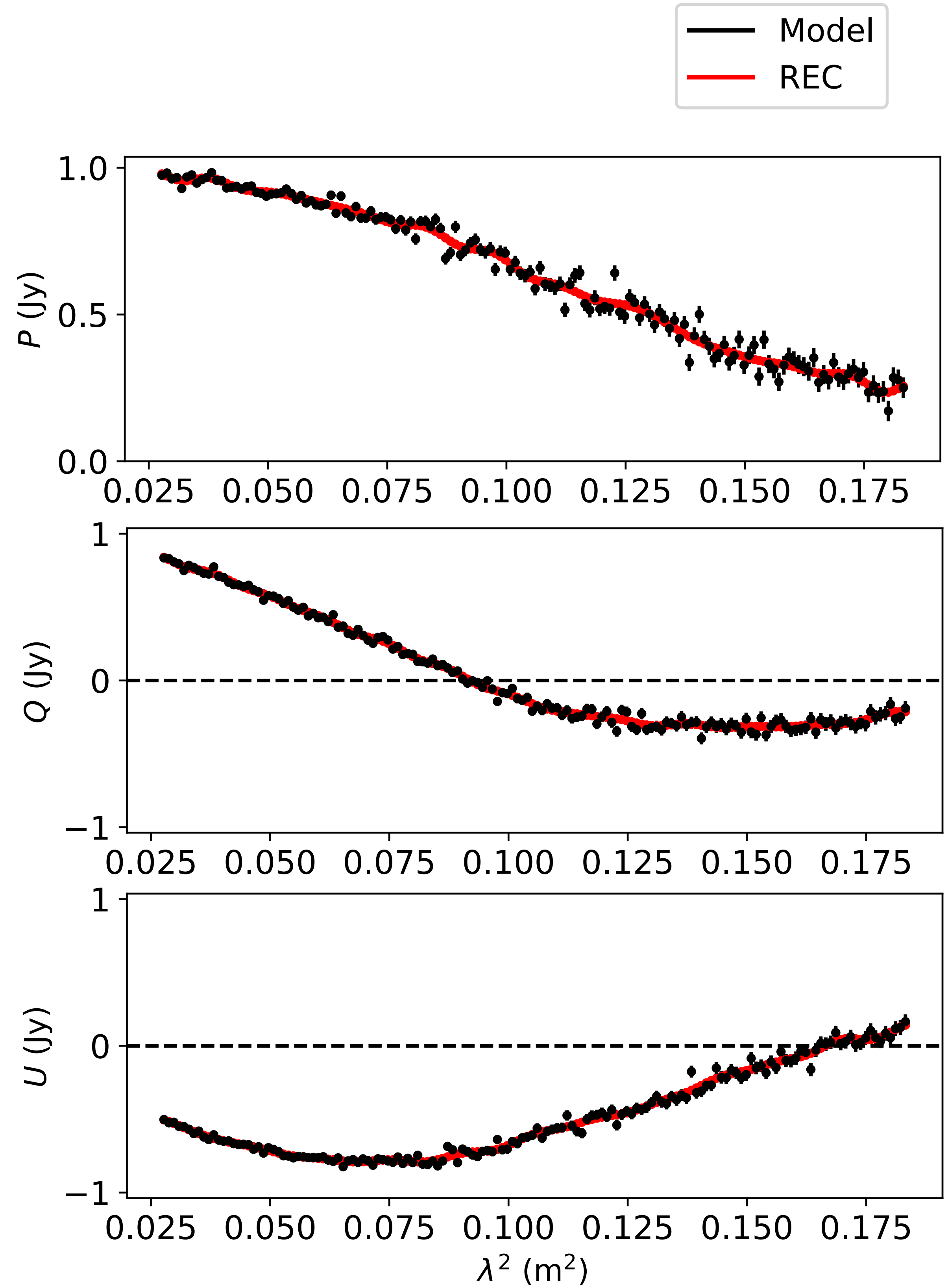
reduced chi-square : 2.31

this coverage can reconstruct sharp structure

Simulation

700-1800MHz





Details

FWHM : 22.27 [rad/m^2]

Λ_l (sparsity) : 0.1

Λ_t (smoothness) : 10

reduced chi-square : 1.90

this coverage can reconstruct smooth structure

Summary

◎Faraday spectrum F(ϕ) shows the 3D distribution of magnetic fields & polarized sources as a function of Faraday depth

◎realistic F(ϕ) is very complicated and delicate, reflect to physical information such as global/random magnetic fields

◎sparse reconstruction can determine an unique solution in an undetermined system if most of the F(ϕ) components are 0

◎sparse reconstruction depends on

- frequency coverage - fix the scale
- regularization coefficients Λ_l , Λ_t - fix the shape of F(ϕ)
- observable numbers
- S/N

◎sparse reconstruction could be powerful method for the recovery technique!