Detecting topological order in the Heisenberg picture
A 1D numerical approach for 2D quantum systems

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Outline

1. Introduction
2. Ribbon operators
3. Optimization problem
4. Numerical results
5. Discussion & Conclusion
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Given access to the ground state:

- **Topological entanglement entropy** $S(\rho_R) = \alpha |\partial R| - \gamma + \mathcal{O}(|R|^{-1})$
  - $\gamma = \log(\sqrt{\sum_c d_c^2})$.
  - Same $\gamma$ for different TQFT (Heisenberg antiferromagnet on the Kagome).
  - $\gamma \neq 0$ with no topological order.

- Entanglement spectrum $\rho_R = e^{-H_{\text{eff}}}$.

- **PEPS description of ground state.**
  - String-like operators that pull through the tensors on the virtual level.

These all require access to the ground state.
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Our contribution

- A numerical method to detect features of a TQFT without actually knowing the ground state!
- Can extract all the topological $S$ matrix elements.
- The numerical problem boils down to 1D DMRG (at the operator level).
- The approach is not rigorous... it works better than it should!
- Perhaps it will fail for more challenging models.
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Anyons

- Point-like excitations that can move freely.
- Topological charge defined by equivalent class of shallow quantum circuits (conservation).
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![Diagram of Anyons](image-url)
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This implies $U_1 U_2 \Pi_{GS} = U_2 U_1 \Pi_{GS}$.

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$U_1$ and $U_2$ are logical operators, i.e., operators which act inside this degenerate ground space.
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Ribbon operators

Topological data from string operators

- Ground space expectation of twist product $U_a \infty U_b \Pi_{GS} = \tilde{S}_{ab} \Pi_{GS}$ reveals (close cousin of) topological $S$-matrix element.
- Can be evaluated efficiently from a shallow circuit representation of $U_{a/b}$ or MPO representation.
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- Preserve the ground state: $[U_a^R, H] \Pi_{GS} = 0$
- Reveal non-trivial topological data $U_a^R U_b^{R'} \Pi_{GS} = \eta U_b^{R'} U_a^R \Pi_{GS}$.
- Be deformable, i.e. changing the location of $R$ should not affect the above.

**Objective function:**

$$C(U_a, U_b, \eta) = \sum_{R \text{ crosses } R'} \| [H, U_a^R] \Pi_{GS} \|^2 + \| [H, U_b^{R'}] \Pi_{GS} \|^2 + \lambda \| U_a^R U_b^{R'} \Pi_{GS} - \eta U_b^{R'} U_a^R \Pi_{GS} \|^2$$

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Numerical approach

- **Vectorize matrices:**
  - \( M = |\phi\rangle\langle\psi| \rightarrow |M\rangle = |\phi\rangle \otimes |\psi\rangle \).
  - \([H, M] \rightarrow (H \otimes I - I \otimes H)|M\rangle\)

- Given \( U^R_b \), topological constraint \( \| U^R_a U^R_b − \eta U^R_b U^R_a \| \) is local:
  - \( \langle U^R_a U^R_b | \tilde{U}^R_{a R^R} | U^R_a \rangle \) for some operator \( \tilde{U}^R_{b R^R} \) supported on \( R \cap R' \) (point).

- When \( H \) is the sum of local terms, Hamiltonian penalty \( \|[H, U^R_a]\|^2 \) becomes an MPO cost function:
  - \( \langle U^R_a | \tilde{H}_R | U^R_a \rangle \) for some MPO \( \tilde{H}_R \) supported on \( R \).

- For fixed \( U^R_b \), objective function is an MPO \( \langle U^R_a | \tilde{O} | U^a \rangle \).
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- To solve for \( U_a \) and \( U_b \), alternate between two independent optimizations.
Vectorize matrices:

- \( M = |\phi\rangle\langle\psi| \rightarrow |M\rangle = |\phi\rangle \otimes |\psi\rangle. \)
- \([H, M] \rightarrow (H \otimes I - I \otimes H)|M\rangle\)

Given \( U_b^{R'} \), topological constraint \( \| U_a^R U_b^{R'} - \eta U_b^{R'} U_a^R \| \) is local:

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2. Ribbon operators
3. Optimization problem
4. Numerical results
5. Discussion & Conclusion
Toric-Ising

\[ H = J \cdot \text{Toric} - \frac{\hbar}{2} \sum_j j(X_j + X_j^\dagger) - \frac{\lambda}{4} \sum_{\langle j,k \rangle} (Z_j + Z_j^\dagger)(Z_k + Z_k^\dagger) \]

\[ -\log_{10} C(R; e^{i\phi}) \]

\[ \mathbb{Z}_5, \{J, h, \lambda\} = \{1, 0, 0\}, \chi = 5, w = 1 \]
Toric-Ising

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\[ \mathbb{Z}_2, \{ J, h, \lambda \} = \{ 1, 0.05, 0 \}, \chi = 5, w = 2 \]
Toric-Ising

\[
H = J \cdot \text{Toric} - \frac{h}{2} \sum_j j(X_j + X_j^\dagger) - \frac{\lambda}{4} \sum_{\langle j,k \rangle} (Z_j + Z_j^\dagger)(Z_k + Z_k^\dagger)
\]

Alternating minimization

\[
\mathbb{Z}_3, \{J, h, \lambda\} = \{1, 0.05, 0\}, \chi = 1, w = 1
\]
$H = -J_x \sum_{j,k \in x\text{-link}} X_j X_k - J_y \sum_{j,k \in y\text{-link}} Y_j Y_k - J_z \sum_{j,k \in z\text{-link}} Z_j Z_k$

$\mathbb{Z}_2$ phase for $0 < |J_x| + |J_y| < J_z$. 
Honeycomb

\[ H = -J_x \sum_{j,k \in \text{x-link}} X_j X_k - J_y \sum_{j,k \in \text{y-link}} Y_j Y_k - J_z \sum_{j,k \in \text{z-link}} Z_j Z_k \]

\( \mathbb{Z}_2 \) phase for \( 0 < |J_x| + |J_y| < J_z \).

\( \{J_x, J_y, J_z\} = \{0.1, 0.1, 1\}, \chi = 5 \)
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$\mathbb{Z}_2$ phase for $0 < |J_X| + |J_Y| < J_Z$.

\[\{J_X, J_Y, J_Z\} = \{0.1, 0.1, 1\}, \ w = 2\]
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\[ H = -J_x \sum_{j,k \in x \text{--link}} X_j X_k - J_y \sum_{j,k \in y \text{--link}} Y_j Y_k - J_z \sum_{j,k \in z \text{--link}} Z_j Z_k \]

$\mathbb{Z}_2$ phase for $0 < |J_x| + |J_y| < J_z$.

\[ \{J_x, J_y, J_z\} = \{J, J, 1\}, \chi = 4, w = 3 \]
Numerical results

Compass model – Not topologically ordered

\[ H = -J_x \sum_{j,k \in \text{x-link}} X_j X_k - J_z \sum_{j,k \in \text{z-link}} Z_j Z_k \]

\[ J_x = J_z, \; \chi = 5 \]
Numerical results

Compass model – Not topologically ordered

\[ H = -J_x \sum_{j,k \in x-\text{link}} X_j X_k - J_z \sum_{j,k \in z-\text{link}} Z_j Z_k \]

Supports vertical and horizontal logical operators,
Compass model – Not topologically ordered

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\[ H = J \cdot \text{Toric} - \frac{h}{2} \sum j(X_j + X_j^\dagger) - \frac{\lambda}{4} \sum_{\langle j,k \rangle} (Z_j + Z_j^\dagger)(Z_k + Z_k^\dagger) \]

\[ \mathbb{Z}_2, \{J, h, \lambda\} = \{1, 0, 0\}, \chi = 5 \]
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Why does it work?

- $\Pi_{GS} U^R_a \Pi_{GS} \Rightarrow U^R_a$ enforce commutation relation on entire spectrum.
- $\Pi_{GS} U^R_a \Pi_{GS} \approx \exp\{-H/\Delta\} U^R_a \exp\{-H/\Delta\}$.
- For a local Hamiltonian, $\exp\{-H/\Delta\}$ maps a ribbon MPO to a (fatter and heavier) ribbon MPO.

If the commutations relations can only be achieved on the low energy sector, given enough width and bond dimension, the minimization problem should output the projected ribbon operator $\Pi_{GS} U^R_a \Pi_{GS}$. 
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It is possible heuristically to learn topological data from a Hamiltonian without having access to the ground state.

Numerically equivalent to 1D DMRG.

Why does it work at all?
- Why can we replace ground-state expectations by operator equalities?
- Does it rely on the structure of excited states being weakly-interacting Anyons?
- For gapped models, the projected ribbon operator should also be a ribbon MPO.

Our numerical benchmarks were for Abelian anyons.
- Can substitute the twist product by group commutator (simpler).
- Any reason this should fail when optimizing the twist product in non-Abelian models?

Can we extract other topological data from these string operators?

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