Dark Matter and Dark Energy or Alternative Theories of Gravity

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For scientific community 2005 was World year of physics
due to Einstein’s papers and the birth of modern physics... and people knew it in advance.
For physical community 2006 was year of astrophysics and cosmology
due to the Nobel prize for CMB discoveries done by J. Mather and G. Smoot .. and people recognized it in the end of 2006...

Plus

Peter Gruber prize on cosmology for J. Mather and COBE team
For astrophysical community 2007 is year of ... 

and people will recognize it after some time...
Outline of the talk

- Discovery of quadrupole anisotropy of CMB in 1992: Relikt-1, COBE etc.
- 1998: Acceleration of the Universe and \( \Lambda \)-term
- Fourth order gravity and its modifications
- Success of the models
- Solar system constraints on \( R^n \) theories
- Conclusions
Because the Earth moves relative to the CMB, a dipole temperature anisotropy of the level of $\Delta T/T = 10^{-3}$ is expected. This was observed in the 1970s (Conklin 1969, Henry 1971, Corey and Wilkinson 1976 and Smoot, Gorenstein and Muller 1977). During the 1970-ties the anisotropies were expected to be of the order of $10^{-2} - 10^{-4}$, but were not observed experimentally. When dark matter was taken into account in the 1980-ties, the predicted level of the fluctuations was lowered to about $10^{-5}$, thereby posing a great experimental challenge.
Info about the Relikt-1

In 1983, the Soviet Union conducted the Relikt-1 experiment aboard the Prognoz-9 satellite in order to pinpoint CMB for the first time in history. This experiment was prepared by the Space Research Institute of the Soviet Academy of Sciences and supervised by Dr. Igor Strukov.

The Prognoz-9, featuring an 8 mm band radiometer with an unprecedentedly high sensitivity of 35 microkelvin per second, was placed into a high apogee orbit with a 400,000 km semi-axis.

The radiometer comprised two megaphone antennas, each with a 50% directional pattern. Both antennas formed a $90^\circ$-degree angle and the same radiometric pattern.

It took the satellite two minutes to rotate along one of its axes. The
The mainframe megaphone antenna was directed along the rotating axis and received radio signals from a preset celestial point. The second antenna conducted a complete scan of the ecliptic plane perpendicular to the rotating axis in two minutes.

Consequently, each element of this ecliptic was scanned several thousand times in a week.

The satellite then changed its spatial orientation and scanned another ecliptic. It took six months to scan the entire celestial sphere.

Computerized data processing and modeling methods were slow at that time. Moreover, the radiometer could not conduct multi-band astronomical observations.

A multi-band experiment would have provided an insight into anisotropy, whereas the single-band experiment left a lot of room for speculation.
A preliminary assessment of a radio signal pattern based on Relikt-1 data produced negative results; however, the temperature variation, i.e. anisotropy of CMB was confirmed many years later.

In 1986, the Space Research Institute’s Academic Council decided to study the anisotropy of CMB as part of the Relikt-2 project. The sensitivity of the equipment had increased fivefold by that time and exceeded that of the Relikt-1 satellite 20 times over.

The Libris spacecraft was scheduled to lift off in 1993-1994, but the launch never took place because of the Soviet Union’s break-up and lack of funding.

The discovery of anisotropy by the Relikt-1 spacecraft was first reported officially in January 1992 at the Moscow astrophysical seminar.
The Relikt Experiment Prognoz 9, launched on 1 July 1983 into a high-apogee (700,000 km) orbit, included the Relikt-1 experiment to investigate the anisotropy of the CMB at 37 GHz, using a Dicke-type modulation radiometer. During 1983 and 1984 some 15 million individual measurements were made (with 10% near the galactic plane providing some 5000 measurements per point). The entire sky was observed in 6 months. The angular resolution was 5.5 degrees, with a temperature resolution of 0.6 mK. The galactic microwave flux was measured and the CMB dipole observed. A quadrupole moment was found between 17 and 95 microKelvin rms, with 90% confidence level. A map of most of the sky at 37 GHz is available.

References: I.A. Strukov, A.A. Brukhanov, D.P. Skulachev and M.V. Sazhin. Pis’ma v Astronomicheskii Zhurnal v.18 (1992), 387 (in Russian, English version: Soviet Astronomy
The Nobel Prize in Physics for 2006 is awarded to John Mather and George Smoot. They have made measurements looking back into the infancy of the Universe and attempted to gain some understanding of the origin of galaxies and stars.

Press Release 3 October 2006

The Nobel Prize in Physics 2006 The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2006 jointly to

John C. Mather (NASA Goddard Space Flight Center, Greenbelt, MD, USA), and George F. Smoot (University of California, Berkeley, CA, USA)

"for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation"
Relikt-1
Абсолютный спектрометр далекой ИК-области

Микроволновые радиометры

Тепловой экран

Сосуд с жидким гелием

антenna

солечные батареи

Эксперимент по изучению рассеянного фонового излучения
Relikt-1
Новые вызовы в астрономии

Гравитационные линзы
Темная материя

Флуктуации реликтового излучения
Темная энергия

Первая карта неоднородностей реликтового излучения (Струков и др., 1992)
Relikt-1 and COBE maps
Figure 5: Relikt-1, COBE, WMAP maps.
Relikt-1 and COBE maps
Trajectories
Figure 7: Relikt-1, COBE etc trajectories.
Submission Dates for Relikt-1 and COBE papers
The Relikt-1 experiment – new results

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SUMMARY
We present new results from reduction of data from the space experiment Relikt-1 (investigation of the anisotropy of the cosmic microwave background at 37 GHz). With 99 per cent confidence, an anomalous signal is detected in a region of area of about 1 sr, centred at RA = 1h30m, Dec. = −10° (l = 150°, b = −70°). The brightness temperature of the signal is \(\Delta T = -71 \pm 43 \mu K\) with 90 per cent confidence, including systematic errors. The nature of the signal cannot be explained by effects of the apparatus or by radio emission of known sources; there are reasons to believe that the signal has a cosmological origin. For a model of cosmological signal with scale-invariant spectrum, i.e. in terms of a power-law spectrum with \(n = 1\), we estimate, for the rms, a quadrupole component of \(6 \times 10^{-6} < \Delta T_2 / T < 3.3 \times 10^{-5}\) with 90 per cent confidence, including systematic errors.

Key words: artificial satellites, space probes – cosmic microwave background – cosmology: observations – large-scale structure of Universe.
Anisotropy of the microwave background radiation

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and P. K. Shternberg State Astronomical Institute, Moscow

(Submitted January 19, 1992)

Pis'ma Astron. Zh. 18, 387–395 (May 1992)

New results from analysis of data on the anisotropy of the background radiation at 37 GHz (spaceborne experiment Relikt 1) are presented. The relative magnitude of the quadrupole component was estimated with 90% confidence for an inflationary perturbation spectrum: $6 \times 10^{-6} < \Delta T_s / T < 3.3 \times 10^{-5}$. An anomaly of the microwave radiation has been found, with 99% confidence, in a region with area $\approx 1$ sr near the point with coordinates $\alpha \approx 1^\circ30'^\circ$ and $\delta \approx -10^\circ$ ($l = 150^\circ$ and $b = -70^\circ$). The magnitude of this anomaly is $\Delta T_s = -71 \pm 43 \mu$K with 90% confidence. We discuss possible sources of the anomaly.

Introduction. This paper is devoted to the results of a follow-on analysis of data from scans of the celestial sphere at 37 GHz in the Relikt 1 spaceborne experiment on the measurement of the background radiation anisotropy (Strukov and Skulachev, 1986; Klypin et al., 1987; Strukov et al., 1988).

The data analysis performed thus far has not revealed a cosmological signal. Only an upper limit on the possible strength of such a signal has been estimated. Such estimates are made primarily by comparing the experimental data, i.e., the signal plus noise, with very accurately determined instrumental noise. In order to make reliable estimates, it is then important to know exactly how the signal and noise are transformed in all units of the experimental apparatus and at all stages of subsequent data processing. In preceding works a number of effects were taken into account approximately. In constructing a map of the celestial sphere we carefully calculated the correlation of separate measurements, and this made it possible to determine more accurately the instrumental noise in the radio map obtained. The noise level was reduced mainly by properly taking into account radiometer sampling by the satellite's telemetry system and by modeling completely the

where $\Delta T_i^2 = (1/4 \pi) \sum_{m=-l}^l (a_m^i)^2$ is in the $l$th spherical harmonic.

For subsequent calculations we employed the signal corresponding to the Harrison–Zel’dovich spectrum for primordial perturbations (Abbott and Wise, 1984; Starobinski, 1983). For such a spectrum

$$\langle \Delta T_i^2 / T^2 \rangle = \pi \epsilon_H^2 (2l+1) / 2l(l+1),$$  \hspace{1cm} (1)

where $T$ is the temperature of the microwave background, $l$ is the number of the spherical harmonic, $\langle \ldots \rangle$ denotes the expectation value, and $\epsilon_H$ is the amplitude of fluctuations at the moment the horizon is crossed.

The signal-to-noise ratio for such a signal can be improved by reducing the high-frequency noise in the map of the experimental data by means of additional smoothing.

Smoothing was performed as follows. The smoothed value $T_{\text{smooth}}$ at a given point on the map was determined from the formula

$$T_{\text{smooth}} = (\sum \sqrt{W_{abc}} A_{abc} \phi) / \sum \sqrt{W_{abc}}.$$
STRUCTURE IN THE COBE\textsuperscript{1} DIFFERENTIAL MICROWAVE RADIOMETER FIRST-YEAR MAPS

G. F. Smoot,\textsuperscript{2} C. L. Bennett,\textsuperscript{3} A. Kogut,\textsuperscript{4} E. L. Wright,\textsuperscript{5} J. Aymon,\textsuperscript{2} N. W. Boggess,\textsuperscript{5} E. S. Cheng,\textsuperscript{3} G. De Amici,\textsuperscript{2} S. Gulkis,\textsuperscript{6} M. G. Hauser,\textsuperscript{3} G. Hinshaw,\textsuperscript{4} P. D. Jackson,\textsuperscript{7} M. Janssen,\textsuperscript{6} E. Kaita,\textsuperscript{7} T. Kelsall,\textsuperscript{3} P. Keegstra,\textsuperscript{7} C. Lineweaver,\textsuperscript{2} K. Loewenstein,\textsuperscript{7} P. Lubin,\textsuperscript{8} J. Mather,\textsuperscript{3} S. S. Meyer,\textsuperscript{9} S. H. Moseley,\textsuperscript{3} T. Murdock,\textsuperscript{10} L. Rokke,\textsuperscript{7} R. F. Silverberg,\textsuperscript{3} L. Tenorio,\textsuperscript{2} R. Weiss,\textsuperscript{9} and D. T. Wilkinson\textsuperscript{11}

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ABSTRACT

The first year of data from the Differential Microwave Radiometers (DMR) on the Cosmic Background Explorer (COBE) show statistically significant (> 7 $\sigma$) structure that is well described as scale-invariant fluctuations with a Gaussian distribution. The major portion of the observed structure cannot be attributed to known systematic errors in the instrument, artifacts generated in the data processing, or known Galactic emission. The structure is consistent with a thermal spectrum at 31, 53, and 90 GHz as expected for cosmic microwave background anisotropy.

The rms sky variation, smoothed to a total 10$^\circ$ FWHM Gaussian, is $30 \pm 5 \mu$K ($\Delta T/T = 11 \times 10^{-6}$) for Galactic latitude $|b| > 20^\circ$ data with the dipole anisotropy removed. The rms cosmic quadrupole amplitude is $13 \pm 4 \mu$K ($\Delta T/T \approx 5 \times 10^{-6}$). The angular autocorrelation of the signal in each radiometer channel and cross-correlation between channels are consistent and give a primordial fluctuation power-law spectrum with index $n = 1.1 \pm 0.5$, and an rms-quadrupole–normalized amplitude of $16 \pm 4 \mu$K ($\Delta T/T \approx 6 \times 10^{-6}$). These features are in accord with the Harrison-Zel’dovich (scale-invariant, $n = 1$) spectrum predicted by models of inflationary cosmology. The low overall fluctuation amplitude is consistent with theoretical predictions of the minimal level gravitational potential variations that would give rise to the observed present day structure.

Subject headings: cosmic microwave background — cosmology: observations

1. INTRODUCTION

The 2.73 K cosmic microwave background (CMB) is one of the most effective probes of the early universe. On large angular scales the CMB contains imprints of the primordial gravitational potential fluctuations (Sachs & Wolfe, 1967), a complete survey of the sky every 6 months while shielding the DMR from terrestrial and solar radiation (Boggess et al. 1992). Smoot et al. (1991) present preliminary results based on 6 months of data and Bennett et al. (1992a) describe the calibration procedure.
Summary on Quadrupole Anisotropy Discovery

The discovery of anisotropy by the Relikt-1 spacecraft was first reported officially in January 1992 at the Moscow astrophysical seminar.

Several months later, George F. Smoot, the head of a similar U.S. project, told a news conference about the discovery of CMB anisotropy by the COBE satellite. The mass media reported this as the main science news of the day.

The projects co-head, John C. Mather, told Newsweek magazine that he knew a lot about the Relikt project, which had been conducted long before the launch of the COBE. He said the project had been one of the first attempts to discover CMB anisotropy, and that to the best of his knowledge, it had proved successful. Mather then congratulated those involved in the Relikt experiment.
He told Newsweek that many researchers had carried out similar projects at that time. He and his team fully acknowledged the achievements of their predecessors, who had obtained many valuable results, but their own results were better.

All this is fine, but according to the Nobel Committees official statement, the prize went to Smoot and Mather for the experimental discovery of the correlation between CMB and its anisotropy. No matter what anyone else may say, Russian scientists were really the first to discover this phenomenon.
Figure 11: COBE’s artist view.
Figure 12: FIRAS sketch.
Figure 13: CMB spectrum.
Latest estimate: $T = 2.725 \pm 0.001$ K

Deviations from blackbody form (Big Bang prediction) are less than 50 parts per million of peak intensity.

New technology could reduce residuals 2 orders of magnitude?

Figure 14: FIRAS residual spectrum.
Figure 15: DMR sketch.
Figure 16: DMR picture.
Sky map from DMR, 2.7 K ± 0.003 K

Doppler Effect of Earth’s motion removed (v/c = 0.001)

Cosmic temperature/density variations at 389,000 years, +/- 0.00003 K

Figure 17: COBE map.
COBE Map of CMB Fluctuations
2.725 K +/- ~ 30 µK rms, 7° beam

Figure 18: COBE map again.
Figure 19: WMAP.
Figure 20: WMAP map.
Figure 21: WMAP power spectrum.
Fig. 9.— top: The first-year ILC map reproduced from Bennett et al. (2003c). middle: The three-year ILC map produced following the steps outlined in §5.2. bottom: The difference between the two (1-yr − 3-yr). The primary reason for the difference is the new bias correction (Figure 8). The low-$l$ change noted in §3 and shown in Figure 3 is also apparent.
Fig. 10.— Galactic foreground removal with spatial templates. All maps in this figure are three-year maps that have had the ILC estimate of the CMB signal subtracted off to highlight the foreground emission. The maps have been degraded to pixel resolution 5, are displayed in Galactic coordinates, and are scaled to ±30 µK. The white contour indicates the perimeter of the Kp2 sky cut, outside of which the template fits were evaluated. The frequency bands Q through W are shown top to bottom. (left) Sky maps prior to the subtraction of the best-fit foreground model (§5.3). (middle) The same sky maps with the first-year template-based model subtracted. Note the high-latitude residuals in the vicinity of the North Polar Spur and around the inner Galaxy due to the use of the Haslam 408 MHz map as a synchrotron template. (right) The same sky maps with the three-year template-based model subtracted. This model substitutes K- and Ka-band data for the Haslam data which produces lower residuals outside the Kp2 sky cut. There are still isolated spots with residual emission of order 30 µK in the vicinity of the Gum Nebula and the Ophiuchus Complex (see Figure 7). Note also that substantial errors (≥30 µK) remain inside the Kp2 cut due to limitations in the template model.
A common opinion: 1998 is the beginning of precise cosmology era

The beginning of the year:
The Universe is opened and $\Omega_m \sim 0.2 - 0.4$ (N. Bachall et al.)

The end of the year:
The Universe is flat $\Omega_m = 0.3, \Omega_\Lambda = 0.7$ (S. Perlmutter et al, A. Riess et al., B. Schmidt et al.)
Solar system constraints on fourth order gravity
(AZ, A. Nucita, F. De Paolis, G. Ingrosso, Phys. Rev. D 74, 107101 (2006))
Is Cosmic Speed-Up Due to New Gravitational Physics?

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We show that cosmic acceleration can arise due to very tiny corrections to the usual gravitational action of general relativity of the form $R^n$, with $n < 0$. This eliminates the need for dark energy, though it does not address the cosmological constant problem. Since a modification to the Einstein-Hilbert action of the form $R^n$, with $n > 0$, can lead to early-time inflation, our proposal provides a unified and purely gravitational origin for the early and late time accelerating phases of the Universe.

\textit{Introduction.} That the expansion of the Universe is currently undergoing a period of acceleration now seems inescapable: it is directly measured from the light-curves of several hundred type Ia supernovae [1, 2, 3], and independently inferred from observations of the cosmic microwave background (CMB) by the WMAP satellite [4] and other CMB experiments [5, 6].

Cosmic speed-up can be accommodated within general relativity by invoking a mysterious cosmic fluid with large negative pressure, dubbed dark energy. The simplest possibility for dark energy is a cosmological constant; unfortunately, the smallest estimates for its value are 55 orders of magnitude too large (for reviews see [7, 8]). This fact has motivated a host of other possibilities, most of which assume a zero cosmological constant, with the dynamical dark energy being associated with a new scalar field [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

However, none of these suggestions is compelling and most have serious drawbacks. Given the challenge of this field [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

The self-accelerating solutions we find can behave like vacuum energy (i.e., $w_{DE} \equiv P_{DE}/\rho_{DE} = -1$), or can lead to power-law acceleration with cosmic scale factor $a \propto t^q$, $q > 1$, and $w_{DE} < -2/3$. We also argue that our model is consistent with existing tests of gravitation theory and that the accelerating phase is consistent with current cosmological observations.

\textit{A Model.} Many authors have considered modifying the Einstein-Hilbert action with terms that become effective in the high-curvature region. Here, however, we explore modifications which become important at extremely low curvatures to explain cosmic speed-up. For definiteness and simplicity we focus on the simplest correction to the Einstein-Hilbert action,

\begin{equation}
S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{\mu^4}{R} \right) + \int d^4x \sqrt{-g} L_M . \tag{1}
\end{equation}

Here $\mu$ is a new parameter with units of [mass], $L_M$ is the Lagrangian density for matter and the reduced Planck mass $M_P \equiv (8\pi G)^{-1/2}$. For work on theories where the
2. gr-qc/0702075  
Title: The cosmological constant as an eigenvalue of f(R)-gravity Hamiltonian constraint  
Authors: S. Capozziello, R. Garattini  

3. astro-ph/0604435  
Title: Gravitational lensing in fourth order gravity  
Authors: S. Capozziello, V.F. Cardone, A. Troisi  

4. astro-ph/0604431  
Title: Cosmological viability of f(R)-gravity as an ideal fluid and its compatibility with a matter dominated phase  
Authors: S. Capozziello, S. Nojiri, S.D. Odintsov, A. Troisi
5. astro-ph/0603522:
Title: Low surface brightness galaxies rotation curves in the low energy limit of $R^n$ gravity: no need for dark matter?
Authors: S. Capozziello, V.F. Cardone, A. Troisi
6. gr-qc/0603071:
Title: Fourth order gravity and experimental constraints on Eddington parameters
Authors: S. Capozziello, A. Stabile, A. Troisi

8. astro-ph/0602349:
Title: Dark energy and dark matter as curvature effects
Authors: S. Capozziello, V.F. Cardone, A. Troisi
Journal-ref: JCAP 0608 (2006) 001

10. hep-th/0512118:
Title: Dark Energy: the equation of state description versus scalar-tensor or modified gravity
Authors: S. Capozziello, S. Nojiri, S.D. Odintsov
15. astro-ph/0508350:
Title: Observational constraints on dark energy with generalized equations of state
Authors: S. Capozziello, V.F. Cardone, E. Elizalde, S. Nojiri, S. D. Odintsov
16. astro-ph/0507545:
Title: PPN-limit of Fourth Order Gravity inspired by Scalar-Tensor Gravity
Authors: S. Capozziello, A. Troisi

17. astro-ph/0507438:
Title: Dark energy exponential potential models as curvature quintessence
Authors: S. Capozziello, V.F. Cardone, E. Piedipalumbo, C. Rubano
Journal-ref: Class.Quant.Grav. 23 (2006) 1205-1216

20. astro-ph/0501426:
Title: Reconciling dark energy models with f(R) theories
Authors: S. Capozziello, V.F. Cardone, A. Troisi
21. astro-ph/0411114:
Title: Higher Order Curvature Theories of Gravity Matched with Observations: a Bridge Between Dark Energy and Dark Matter Problems
Authors: S. Capozziello, V.F. Cardone, S. Carloni, A. Troisi
Comments: 8 pages, Proceedings of XVI SIGRAV conference, 13-16 September, Vietri (Italy)
24. astro-ph/0410135:
Title: f(R) theories of gravity in Palatini approach matched with observations
Authors: S. Capozziello, V.F. Cardone, M. Francaviglia

25. gr-qc/0410046:
Title: Cosmological dynamics of $R^n$ gravity
Authors: S. Carloni, P. K. S. Dunsby, S. Capozziello, A. Troisi
Journal-ref: Class.Quant.Grav. 22 (2005) 4839-4868
1. astro-ph/0607639:
Title: Dark matter and dark energy as a effects of Modified Gravity
Authors: Andrzej Borowiec, Wlodzimierz Godlowski, Marek Szydlowski
Comments: Lectures given at 42nd Karpacz Winter School of Theoretical Physics: Ladek, Poland, 6-11 Feb 2006

2. astro-ph/0602526:
Title: Accelerated Cosmological Models in Modified Gravity tested by distant Supernovae SNIa data
Authors: Andrzej Borowiec, Wlodzimierz Godlowski, Marek Szydlowski
Astrophysical observations are pointing out huge amounts of dark matter and dark energy needed to explain the observed large scale structures and cosmic accelerating expansion. Up to now, no experimental evidence has been found, at fundamental level, to explain such mysterious components. The problem could be completely reversed considering dark matter and dark energy as shortcomings of General Relativity and claiming for the correct theory of gravity as that derived by matching the largest number of observational data. As a result, accelerating behavior of cosmic fluid and rotation curves of spiral galaxies are reproduced by means of curvature effects.
**TABLE I**: Best fit values of the model parameters from maximizing the joint likelihood function $\mathcal{L}(\beta, \log r_c, \varphi_0)$. We also report the value of $\Upsilon_\star$, the $\chi^2$/dof for the best fit parameters (with $dof = N - 3$ and $N$ the number of datapoints) and the root mean square $\sigma_r$ of the fit residuals.

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<th>Id</th>
<th>$\beta$</th>
<th>$\log r_c$</th>
<th>$\varphi_0$</th>
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<td>0.526</td>
<td>-1.87</td>
<td>0.69</td>
<td>3.14</td>
<td>0.47/8</td>
<td>0.21</td>
</tr>
<tr>
<td>UGC 10310</td>
<td>0.608</td>
<td>-1.61</td>
<td>0.65</td>
<td>1.04</td>
<td>3.93/13</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Infinite thin and circularly symmetric disk with surface density $\Sigma(R) = \Upsilon_\star I_0 \exp(-R/R_d)$ where the central surface luminosity $I_0$ and the disk scalelength $R_d$ are obtained from fitting to the stellar photometry. The gas surface density has been obtained by interpolating the data over the range probed by HI measurements and ex-

![FIG. 1: Best fit theoretical rotation curve superimposed to the data for the LSB galaxy NGC 4455 (left) and NGC 5023 (right). To better show the effect of the correction to the Newtonian gravitational potential, we report the total rotation curve $v_c(R)$ (solid line), the Newtonian one (short dashed) and the corrected term (long dashed).](image-url)
We investigate the possibility that the observed flatness of the rotation curves of spiral galaxies is not an evidence for the existence of dark matter haloes, but rather a signal of the breakdown of General Relativity. To this aim, we consider power-law fourth order theories of gravity obtained by replacing the scalar curvature $R$ with $f(R) = f_0 R^n$ in the gravity Lagrangian. We show that, in the low energy limit, the gravitational potential generated by a pointlike source may be written as $\Phi(r) \propto r^{-1} \left[1 + (r/r_c)^\beta\right]$ with $\beta$ a function of the slope $n$ of the gravity
Lagrangian and $r_c$ a scalelength depending on the gravitating system properties. In order to apply the model to realistic systems, we compute the modified potential and the rotation curve for spherically symmetric and for thin disk mass distributions. It turns out that the potential is still asymptotically decreasing, but the corrected rotation curve, although not flat, is higher than the Newtonian one thus offering the possibility to fit rotation curves without dark matter. To test the viability of the model, we consider a sample of 15 low surface brightness (LSB) galaxies with combined HI and H$\alpha$ measurements of the rotation curve extending in the putative dark matter dominated region. We find a very good agreement between the theoretical rotation curve and the data using only stellar disk and interstellar gas.
Figure 1. Contour plots for $v_c(R_d)$ in the planes $(\beta, \log r_c)$, $(\beta, f_g)$ (middle), $(\log r_c, f_g)$ (right) with $r_c$ in kpc. The contours are plotted for $v_c(R) = k \times v_{fid}$ with $k$ from 0.7 to 1.3 in steps of 0.1 and $v_{fid} = v_c(R_d)$ for the model with $(\beta, \log r_c, f_g) = (0.61, -2.13, 0.65)$. Upper panels refer to a point-like system with total mass $m = \Upsilon \times L_d + M_{HI}$, with $L_d$ the total disk luminosity, $M_{HI}$ the gas mass and $\Upsilon$, given by Eq. (39). Lower panels refer to the extended case using as default parameters those of UGC 10310. In each panel, the remaining parameter is set to its fiducial value. Note that similar plots are obtained for values of $R$ other than $R_d$. 

**LSB rotation curves and $R^n$ gravity theories**
Figure 2. Some illustrative examples of simulated rotation curves (smoothing the data for convenience) with overplotted the input theoretical rotation curve (solid line) and the best fit one (short dashed line).

Figure 3. Contours of equal $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ projected on the three planes $(\beta, \log r_c)$, $(\beta, f_g)$, $(\log r_c, f_g)$ for the case of the simulation in the top right panel of Fig. 2 with $r_c$ in kpc. In each panel, the remaining parameter is set to its best fit value. The three contours individuate the 1, 2 and 3$\sigma$ confidence ranges. Open contours mean that no constraints may be obtained.
Figure 2. Some illustrative examples of simulated rotation curves (smoothing the data for convenience) with overplotted the input theoretical rotation curve (solid line) and the best fit one (short dashed line).

Figure 3. Contours of equal $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ projected on the three planes ($\beta, \log r_c$), ($\beta, f_g$), ($\log r_c, f_g$) for the case of the simulation in the top right panel of Fig. 2 with $r_c$ in kpc. In each panel, the remaining parameter is set to its best fit value. The three contours individuate the 1, 2 and 3$\sigma$ confidence ranges. Open contours mean that no constraints may be obtained.
erasing the best fit values for the 11 successfully fitted gala x-

Table 2 for the values of the best fit parameters. A case by c ase discussion is presented in the Appendix A.

Both these values are typical of LSB galaxies thus suggest-

Figure 5. Best fit curves superimposed to the data for the sample of 15 LSB galaxies considered. See Table 1 for details on the galaxie s

Figure 28: Rotation curves.

$\langle f \rangle \approx g$

S. Capozziello et al.

$\frac{v_c}{\text{km/s}} \approx \frac{M}{L}$

$\frac{v_c}{\text{km/s}} \approx \frac{M}{L}$

NGC 2366

UGC 3371

UGC 1230

DDO 185

IC 2233

NGC 4455

UGC 1281

UGC 4173

UGC 3137

UGC 4325

NGC 3274

UGC 10310

DDO 189

NGC 2473

Figure 28: Rotation curves.

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NGC 4455

UGC 1281

UGC 4173

UGC 3137

UGC 4325

NGC 3274

UGC 10310

DDO 189

NGC 2473

Figure 28: Rotation curves.
this procedure is not completely correct since the errors are not Gaussian so that they are likely to be overestimated (especially when giving rise to unphysical negative lower limits for $\Upsilon_{\star}^{\Upsilon 1230}$ and DDO 189, the fitted within 1
Eq.(39) for the best fit into account. Moreover, we have assumed the same§§ any molecular gas in the gas budget. Although this is typi-
posed scenario would fail for these two galaxies. Deferring significant bias in the estimated $M/L$ Should
some suitably chosen population synthesis models predict the values of $\Upsilon_{\star}$.

Table 2. Best fit values of the model parameters from minimizing $\chi^2(\beta, \log r_c, f_g)$ with $\beta = 0.817$ corresponding to $n = 3.5$ as obtained from the best fit to the SNeIa data with only baryonic matter. We report 1\(\sigma\) (2\(\sigma\)) confidence ranges on the fitting parameters computed by projecting on the (log $r_c, f_g$) axes the contours $\Delta \chi^2 = 1$ ($\Delta \chi^2 = 4$). The best fit stellar $M/L$ ratio $\Upsilon_{\star}$ has been obtained evaluating Eq.(39) for the best fit $f_g$, while the uncertainty is obtained by usual propagation of errors symmetrizing the 1\(\sigma\) range of $f_g$. Note that this procedure is not completely correct since the errors are not Gaussian so that they are likely to be overestimated (especially when giving rise to unphysical negative lower limits for $\Upsilon_{\star}$). We also give $\chi^2/d.o.f.$ for the best fit model.

<table>
<thead>
<tr>
<th>Id</th>
<th>$\log r_c$ 1(\sigma)</th>
<th>$\log r_c 2\sigma$</th>
<th>$f_g$ 1(\sigma)</th>
<th>$f_g 2\sigma$</th>
<th>$\Upsilon_{\star}$</th>
<th>$\chi^2/dof$</th>
</tr>
</thead>
<tbody>
<tr>
<td>UGC 1230</td>
<td>(−0.59, −0.13)</td>
<td>(−0.78, −0.05)</td>
<td>0.15</td>
<td>(0.13, 0.18)</td>
<td>(0.10, 0.21)</td>
<td>15.9 ± 3.1 ± 7.1</td>
</tr>
<tr>
<td>UGC 1281</td>
<td>(−2.26, −1.95)</td>
<td>(−2.38, −1.76)</td>
<td>0.47</td>
<td>(0.37, 0.56)</td>
<td>(0.29, 0.66)</td>
<td>1.36 ± 0.53 ± 1.04</td>
</tr>
<tr>
<td>UGC 3137</td>
<td>(−1.67, −1.63)</td>
<td>(−1.73, −0.60)</td>
<td>0.61</td>
<td>(0.59, 0.63)</td>
<td>(0.57, 0.64)</td>
<td>12.0 ± 0.9 ± 1.8</td>
</tr>
<tr>
<td>UGC 3371</td>
<td>(−1.78, −1.52)</td>
<td>(−2.16, −1.21)</td>
<td>0.40</td>
<td>(0.28, 0.54)</td>
<td>(0.20, 0.67)</td>
<td>3.5 ± 1.9 ± 3.8</td>
</tr>
<tr>
<td>UGC 4173</td>
<td>(−0.74, −0.16)</td>
<td>(−1.39, 0.55)</td>
<td>0.36</td>
<td>(0.26, 0.49)</td>
<td>(0.20, 0.65)</td>
<td>8.9 ± 5.1 ± 11.3</td>
</tr>
<tr>
<td>UGC 4325</td>
<td>(−2.81, −2.62)</td>
<td>(−3.07, −2.36)</td>
<td>0.69</td>
<td>(0.55, 0.80)</td>
<td>(0.40, 0.89)</td>
<td>5.1 ± 0.33 ± 0.69</td>
</tr>
<tr>
<td>NGC 2366</td>
<td>(−0.47, 1.05)</td>
<td>(−0.77, 1.25)</td>
<td>0.18</td>
<td>(0.17, 0.20)</td>
<td>(0.15, 0.23)</td>
<td>14.4 ± 1.9 ± 4.4</td>
</tr>
<tr>
<td>IC 2233</td>
<td>(−2.12, −1.96)</td>
<td>(−2.19, −1.87)</td>
<td>0.60</td>
<td>(0.55, 0.64)</td>
<td>(0.50, 0.68)</td>
<td>1.56 ± 0.29 ± 0.60</td>
</tr>
<tr>
<td>NGC 3274</td>
<td>(−2.09, −2.03)</td>
<td>(−2.19, −1.98)</td>
<td>0.49</td>
<td>(0.47, 0.52)</td>
<td>(0.44, 0.55)</td>
<td>2.89 ± 0.30 ± 0.60</td>
</tr>
<tr>
<td>NGC 4395</td>
<td>(−0.25, −0.05)</td>
<td>(−0.69, 0.23)</td>
<td>0.09</td>
<td>(0.09, 0.10)</td>
<td>(0.08, 0.11)</td>
<td>12.1 ± 1.6 ± 2.5</td>
</tr>
<tr>
<td>NGC 4455</td>
<td>(−2.36, −2.30)</td>
<td>(−2.46, −2.24)</td>
<td>0.85</td>
<td>(0.82, 0.87)</td>
<td>(0.79, 0.89)</td>
<td>0.38 ± 0.08 ± 0.17</td>
</tr>
<tr>
<td>NGC 5023</td>
<td>(−2.52, −2.46)</td>
<td>(−2.63, −2.40)</td>
<td>0.52</td>
<td>(0.49, 0.55)</td>
<td>(0.46, 0.58)</td>
<td>1.02 ± 0.12 ± 0.26</td>
</tr>
<tr>
<td>DDO 185</td>
<td>(−2.74, −2.52)</td>
<td>(−2.87, −2.10)</td>
<td>0.94</td>
<td>(0.71, 0.97)</td>
<td>(0.41, 1.00)</td>
<td>0.12 ± 0.49 ± 1.14</td>
</tr>
<tr>
<td>DDO 189</td>
<td>(−1.82, −1.47)</td>
<td>(−2.00, −1.24)</td>
<td>0.53</td>
<td>(0.43, 0.62)</td>
<td>(0.35, 0.72)</td>
<td>6.44 ± 2.52 ± 4.90</td>
</tr>
<tr>
<td>UGC 10310</td>
<td>(−1.76, −1.56)</td>
<td>(−2.05, −1.34)</td>
<td>0.56</td>
<td>(0.46, 0.65)</td>
<td>(0.37, 0.74)</td>
<td>1.55 ± 0.60 ± 1.16</td>
</tr>
</tbody>
</table>

LSB rotation curves and $R^a$ gravity theories

Table 2. Best fit values of the model parameters from minimizing $\chi^2(\beta, \log r_c, f_g)$ with $\beta = 0.817$ corresponding to $n = 3.5$ as obtained from the best fit to the SNeIa data with only baryonic matter. We report 1\(\sigma\) (2\(\sigma\)) confidence ranges on the fitting parameters computed by projecting on the (log $r_c, f_g$) axes the contours $\Delta \chi^2 = 1$ ($\Delta \chi^2 = 4$). The best fit stellar $M/L$ ratio $\Upsilon_{\star}$ has been obtained evaluating Eq.(39) for the best fit $f_g$, while the uncertainty is obtained by usual propagation of errors symmetrizing the 1\(\sigma\) range of $f_g$. Note that this procedure is not completely correct since the errors are not Gaussian so that they are likely to be overestimated (especially when giving rise to unphysical negative lower limits for $\Upsilon_{\star}$). We also give $\chi^2/d.o.f.$ for the best fit model.

Indeed, as a cross check, we have used the Bell & de Jong (2001) formulae with the colors available in the NED database§§ obtaining values of $\Upsilon_{\star}$ typically much smaller than 1. This is in contrast with the usual claim that $M/L \simeq 1.4$ for LSB galaxies (de Blok & Bosma 2002), while some suitedly chosen population synthesis models predict literally a good assumption, it is worth noting that our modified potential may increase the contribution to the total rotation curve of any mass element so that it is possible that the missing matter in UGC 1230 and DDO 189 is represented by unaccounted molecular gas. However, even excluding these two galaxies, we end up with a conservative estimate of 10 over
Gravitational lensing is investigated in the weak field limit of fourth order gravity in which the Lagrangian of the gravitational field is modified by replacing the Ricci scalar curvature $R$ with an analytical expression $f(R)$. Considering the case of a pointlike lens, we study the behavior of the deflection angle in the case of power law Lagrangians, i.e. with $f(R) = f_0 R^n$. In order to investigate possible detectable signatures, the position of the Einstein ring and the solutions of the lens equation are evaluated considering the change with respect to the standard case. Effects on the amplification of the images and the Paczynski curve in microlensing experiments are also estimated.
depend in a complicated way on both the theory parameters ($\beta, \log \tilde{v}_1$) and the minimum impact parameter $\vartheta_0$. Although a general rule cannot be extracted from the plots, it is interesting to note, however, that, for some combination of the parameters, the microlensing lightcurve may be also distorted with respect to the Paczynski one. Nevertheless, the typical signature of microlensing events (symmetry, uniqueness of the bump and acromaticity) are preserved.

To estimate quantitatively the deviations from the Paczynski lightcurve, we plot in Fig. 3 the quantity $\Delta A_{\log} = 1$ as function of the time $t$ for the same values on the PPN parameters. Moreover, in the weak field limit, they give rise to a modified gravitational potential that makes it possible to fit the LSB rotation curves without the need of any dark matter halo.

Having successfully passed this impressive set of observational tests, the proposed modification to Einstein general relativity worths to be further investigated considering its effects on gravitational lensing. As a first step, we have derived an analytical expression for the deflection angle of a pointlike lens since this is the basic ingredient for a generalization to the case of extended systems (such as galaxies and clusters of galaxies). Because...
Microlensing scales are few A.U.

Therefore, there are significant deviations from the Newtonian law in the Solar system.
Recent supernovae of type Ia measurements and other astronomical observations suggest that our universe is in accelerating phase of evolution at the present epoch. While a dark energy of unknown form is usually proposed as the most feasible mechanism for the acceleration, there are appears some alternative conception that some effects arising from generalization of Einstein equation can mimic dark energy through a modified Friedmann equation. In this work we investigate some observational constraints
on modified Friedmann equation obtained from generalized Lagrangian $\mathcal{L} \propto R^n$ in minimal coupling with matter in Palatini formalism. We mainly concentrate on the constraints of model parameters from distant supernovae but other constraint from baryon oscillation prior is also considered. We obtain the confidence levels on two additional model parameter $(n, \Omega_{m,0})$. We conclude that the FRW model of First-Order Non-linear gravity survives several observational test like SNIa observation and baryon oscillation peaks. We find preferred value of $\Omega_{m,0} \simeq 0.3$ from combined analysis of supernovae data and baryon oscillation peak. For deeper statistical analysis we apply Akaike and Bayesian information criteria of model selection for comparison prediction of the model with prediction of concordance $\Lambda$CDM model.
TABLE I: The flat non-linear gravity model with $w = 0$. Results of statistical analysis performed on the Astier versus the Gold Riess samples of SNIa obtained from $\chi^2$ best-fit. We separately analysed the case $n > 3$ and $n < 3$.

<table>
<thead>
<tr>
<th>sample</th>
<th>$\Omega_{m,0}$</th>
<th>$\Omega_{\text{nonl},0}$</th>
<th>$n$</th>
<th>$M$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.35</td>
<td>&lt; 0.01</td>
<td>3.001</td>
<td>15.975</td>
<td>180.7</td>
</tr>
<tr>
<td>$n &lt; 3$</td>
<td>0.89</td>
<td>0.23</td>
<td>2.13</td>
<td>15.975</td>
<td>181.5</td>
</tr>
<tr>
<td>$n &gt; 3$</td>
<td>0.35</td>
<td>&lt; 0.01</td>
<td>3.001</td>
<td>15.975</td>
<td>180.7</td>
</tr>
<tr>
<td>Astier</td>
<td>0.01</td>
<td>-1.47</td>
<td>3.11</td>
<td>15.785</td>
<td>108.7</td>
</tr>
<tr>
<td>$n &lt; 3$</td>
<td>0.98</td>
<td>0.08</td>
<td>2.59</td>
<td>15.785</td>
<td>108.9</td>
</tr>
<tr>
<td>$n &gt; 3$</td>
<td>0.01</td>
<td>-1.47</td>
<td>3.11</td>
<td>15.785</td>
<td>108.7</td>
</tr>
</tbody>
</table>

TABLE II: The flat non-linear gravity cosmological model ($w = 0$). The values of the parameters obtained from marginal PDFs calculated on the Astier versus the Gold Riess samples. Because of the singularity at $n = 3$ we separately analyze the cases $n > 3$ and $n < 3$.

<table>
<thead>
<tr>
<th>sample</th>
<th>$\Omega_{m,0}$</th>
<th>$\Omega_{\text{nonl},0}$</th>
<th>$n$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.01</td>
<td>0.26</td>
<td>2.11</td>
<td>15.955+$^{+0.03}_{-0.03}$</td>
</tr>
<tr>
<td>$n &lt; 3$</td>
<td>1.00</td>
<td>0.26</td>
<td>2.11</td>
<td>15.955+$^{+0.03}_{-0.03}$</td>
</tr>
<tr>
<td>$n &gt; 3$</td>
<td>0.01</td>
<td>-0.01</td>
<td>3.001</td>
<td>15.955+$^{+0.03}_{-0.03}$</td>
</tr>
<tr>
<td>Astier</td>
<td>0.01</td>
<td>0.09</td>
<td>2.56</td>
<td>15.785+$^{+0.03}_{-0.03}$</td>
</tr>
<tr>
<td>$n &lt; 3$</td>
<td>1.00</td>
<td>0.09</td>
<td>2.56</td>
<td>15.785+$^{+0.03}_{-0.03}$</td>
</tr>
<tr>
<td>$n &gt; 3$</td>
<td>0.01</td>
<td>-0.01</td>
<td>3.01</td>
<td>15.785+$^{+0.03}_{-0.03}$</td>
</tr>
</tbody>
</table>

the Gold sample. In Figure 4 we present PDF obtained with the Astier sample for the parameters $\Omega_{m,0}$ and $n$ for non-linear gravity model, (case $n < 3$ marginalised over the rest of parameters). Please note that from Fig. 4 we obtain a very weak dependence of PDF on the matter density parameter if only $\Omega_{m,0} > 0.05$. 

68
FIG. 5: The flat non-linear gravity model \((w = 0, n < 3)\). Confidence levels on the \((\Omega_{m,0}, n)\) plane, marginalised over \(M\), obtained from SNIa Astier sample.
FIG. 6: The flat non-linear gravity model \( (w = 0, n < 3) \). Confidence levels on the \((\Omega_{m,0}, n)\) plane obtained from baryon oscillation peaks.

Please note that both information criteria make no absolute sense and only the relative values between different models are physically interesting. For the BIC a difference of 2 is treated as a positive evidence (6 as a strong evidence) against the model with larger value of BIC [33, 34]. Therefore one can order all models, which belong to the ensemble of dark energy models, following the AIC and BIC values. If we do not find any positive evidence from information criteria, the models are treated as identical, while eventually additional parameters are treated as not significant.
FIG. 7: The flat non-linear gravity model \((w = 0, n < 3)\). Common confidence levels on the plane \((\Omega_{m, \rho}, n)\) obtained from SNIa Astier sample and baryon oscillations.
We explain the effect of dark matter (flat rotation curve) using modified gravitational dynamics. We investigate in this context a low energy limit of generalized general relativity with a nonlinear Lagrangian $\mathcal{L} \propto R^n$, where $R$ is the (generalized) Ricci scalar and $n$ is parameter estimated from SNIa data. We estimate parameter $\beta$ in modified gravitational potential $V(r) \propto -\frac{1}{r}(1 + (\frac{r}{r_c})^\beta)$. Then we compare value of $\beta$ obtained from SNIa data with $\beta$ parameter evaluated from the best fitted rotation curve. We find $\beta \simeq 0.7$ which becomes in good agreement with an observation of
spiral galaxies rotation curve. We also find preferred value of $\Omega_{m,0}$ from the combined analysis of supernovae data and baryon oscillation peak. We argue that although amount of "dark energy" (of non-substantial origin) is consistent with SNIa data and flat curves of spiral galaxies are reproduces in the framework of modified Einstein’s equation we still need substantial dark matter. For comparison predictions of the model with predictions of the $\Lambda$CDM concordance model we apply the Akaike and Bayesian information criteria of model selection.
We present an analysis of a devised sample of Rotation Curves (RCs), aimed at checking the consequences of a modified f(R) gravity in galactic scales. Originally motivated by the the dark energy mystery, this theory may serve as a possibility of explaining the observed non-Keplerian profiles of galactic RCs in terms of a break-down of the Einstein General Relativity. We show that in general the power-law f(R) version could fit well the observations with reasonable values for the mass model parameters, encouraging further investigation on $R^n$ gravity from both observational and theoretical points of view.
direct:

\[ V_c^2(r) = r \frac{d}{dr} \phi_c(r) = -2^\beta r_c^{-\beta} \pi \alpha (\beta - 1) G I(r), \]  

where the integral is defined as

\[ I(r) \equiv \int_0^\infty dr' r' \left[ \frac{1}{2} \right] k^{3-\beta} \Sigma(r') F(r), \]  

with \( F(r) \) written in terms of confluent hyper-geometric function: \( F(r) \equiv 2(r + r') 2F_1[\frac{1}{2}, \frac{1}{2}; 1; k^2] + [(k^2 - 2)r' + k^2 r] 2F_1[\frac{3}{2}, \frac{3-\beta}{2}, 2, k^2]. \)

The total circular velocity is the sum of each squared contribution:

\[ V^2(r) = V_{N,stars}^2 + V_{N,\text{gas}}^2 + V_{c,\text{stars}}^2 + V_{c,\text{gas}}^2. \]  

We have checked that the resulting Newtonian circular velocity contribution of the stars coincides, as it should be, with the standard formula

\[ V_{N,\text{stars}}^2(r) = (GM_D/2R_D) x^2 B(x/2), \]

where \( x = r/R_D \), \( G \) is the gravitational constant and the quantity \( B = I_0 K_0 - I_1 K_1 \) is a combination of Bessel functions (Freeman 1970).

In a first step, the RCs are \( \chi^2 \) best-fitted with the following free parameters: the slope (\( \beta \)) and the scale length (\( r_c \)) of the theory, and the gas mass fraction (\( f_{\text{gas}} \)) related to the disk mass simply by \( M_D = M_{\text{gas}}(1-f_{\text{gas}})/f_{\text{gas}} \). From the results of these fits we get a mean value of \( \beta = 0.7 \pm 0.3 \). Then we redo the best-fit fixing the CCT slope parameter at its mean value (\( n = 2.2 \)). Notice that in a previous paper (Capozziello et al. 2006), CCT obtained the mean value of \( \beta = 0.58 \pm 0.15 \), perfectly compatible with our result. This parameter however, according to CCT, is well constrained.

Table 2. Parameters of the mass model of the analyzed galaxies for \( \beta = 0.7 \) (\( n = 2.2 \)). \( \Upsilon_* \) is the mass-to-light ratio in units of \((M_\odot/L_\odot)\), \( r_c \) is the scale length CCT parameter in \( 10^{-2} \) kpc, \( f_{\text{gas}} \) is the gas fraction in %, \( M_D \) is the total mass of the disk in \( 10^9 M_\odot \) and at the end is the \( \chi^2_{\text{red}} \). The galaxies are ordered as in Table 1.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>( \Upsilon_* )</th>
<th>( r_c )</th>
<th>( f_{\text{gas}} )</th>
<th>( M_D )</th>
<th>( \chi^2_{\text{red}} )</th>
</tr>
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<tr>
<td>DDO 47</td>
<td>0.2</td>
<td>0.5</td>
<td>96±1</td>
<td>0.01</td>
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<td>ESO 116-G12</td>
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<td>5±1</td>
<td>50</td>
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<td>1.2</td>
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<td>4.8</td>
<td>41±7</td>
<td>2.8±0.2</td>
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<tr>
<td>ESO 287-G13</td>
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<td>25±1</td>
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<td>59±4</td>
<td>18±1</td>
<td>47</td>
<td>0.9</td>
</tr>
<tr>
<td>UGC 8017</td>
<td>0.4</td>
<td>1±1</td>
<td>-</td>
<td>9.1±0.3</td>
<td>5.2</td>
</tr>
<tr>
<td>M 31</td>
<td>7.1</td>
<td>153±19</td>
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<td>180±70</td>
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</tr>
<tr>
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<td>14±1</td>
<td>-</td>
<td>74±3</td>
<td>5</td>
</tr>
<tr>
<td>UGC 10981</td>
<td>7.6</td>
<td>( \sim 10^{13} )</td>
<td>-</td>
<td>460±200</td>
<td>4.9</td>
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</table>

reason it has been neglected in the analysis. Values of the scale length parameter (\( r_c \)) are smaller for the less massive galaxies and bigger for the more massive ones. We obtained a Newtonian fit for UGC 10981, see the exceedingly large value for \( r_c \).

5 DISCUSSIONS AND CONCLUSIONS

We have investigated the possibility of fitting the RCs of spirals with a power-law fourth order theory of gravity, with...
Figure 1. Best-fit curves superimposed to the data for the sample of 9 galaxies considered. The solid line represents the best-fit mass model, the dotted and the dashed lines are those of, respectively, the Newtonian and the correction contributions. Below the RCs, we plot the residuals ($V_{\text{obs}} - V_{\text{model}}$). See Table 1 for details on the galaxies and Table 2 for the values of the best-fit parameters.
Fits are almost perfect and all things are very smooth on the paper but...
people have forgotten (or wanted to skip) the Solar system...
Since many years different alternative approaches to gravity have been proposed in the literature such as MOND scalar-tensor, Yukawa-like corrected gravity theories and so on. Very recently, it has been proposed by Capozziello et al. (2006) in the framework of higher order theories of gravity – also referred to as $f(R)$ theories – a modification of the gravity action with the form

$$A = \int d^4x \sqrt{-g}[f(R) + \mathcal{L}_m], \quad (1)$$

where $f(R)$ is a generic function of the Ricci scalar curvature and $\mathcal{L}_m$ is the standard matter Lagrangian. For example, if $f(R) = R + 2\Lambda$ the theory coincides with General Relativity (GR) with the $\Lambda$ term. In particular, Capozziello et al. (2006) considered power law function $f(R)$ theories of the form $f(R) = f_0 R^n$. As a result, in the weak field limit, the gravitational
potential is found to be

\[ \Phi(r) = -\frac{Gm}{2r} \left[ 1 + \left( \frac{r}{r_c} \right)^\beta \right] , \quad (2) \]

where

\[ \beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}. \quad (3) \]
Figure 37: The parameter $\beta$ as a function of $n$ for fourth order gravity.
The dependence of the $\beta$ parameter on the $n$ power is shown in Fig. 37. Of course, for $n \to \infty$ it follows $\beta \to 1$, while for $n = 1$ the parameter $\beta$ reduces to zero and the Newtonian gravitational field is recovered. On the other hand, while $\beta$ is a universal parameter, $r_c$ in principle is an arbitrary parameter, depending on the considered system and its typical scale. Consider, for example, the Sun as the source of the gravitational field and the Earth as the test particle. Since Earth velocity is $\simeq 30 \text{ km s}^{-1}$, it has been found that the parameter $r_c$ varies in the range $\simeq 1 - 10^4$ AU.

Once $r_c$ and $\beta$ has been fixed, Capozziello et al. (2006) used them to study deviations from the standard Paczynski light curve for gravitational microlensing (Paczynski (1986)) and claimed that the implied deviation can be measured. It is clear that for gravitational microlensing one could detect observational differences between GR and an alternative theory (the fourth order gravity in particular), so that one should have different potentials at the scale $R_E$ (the Einstein radius) of the gravitational microlensing. For
the Galactic microlensing case $R_E$ is about 1 AU. This is a reason why the authors Capozziello et al. (2006) have selected $r_c$ at a level of astronomical units to obtain observable signatures for non-vanishing $\beta$. The aim of the present paper is to show that solar system data (light bending and planetary periods) put extremely strong constraints on both $r_c$ and $\beta$ parameters making this alternative theory of gravity not so attracting.

As stated above, we now discuss some observational consequences of the fourth order gravity, depending on the choice of the parameters $\beta$ and $r_c$.

A constraint on the proposed theory can be derived by considering the light bending effect in the Sun limb. It is well-known that in the parameterized post-Newtonian formalism the bending angle through which a electromagnetic light ray from a distant source is deflected by a body
with mass $m$ is
\[
\theta = \frac{(1 + \gamma) G m}{c^2 b} (1 + \cos \phi),
\]
where $b$ is the impact parameter, $\phi$ is the solar elongation angle (between the Sun and the source as viewed from Earth) and $\gamma$ is the post-Newtonian parameter. For GR, $\gamma = 1$ and for light rays at Sun’s limb, $\theta_{GR} = 1.75''$. Recently, Shapiro et al. (2004) measured the bending angles for distant compact sources and concluded that light bending angles follow GR with a very high precision ($\gamma = 0.9998 \pm 0.0004$).

In other words, this means that the deflection of the light path is well described by the GR theory. In particular, as radio observations of distant sources have shown the observed and expected bending angles are related by $\theta_{obs} = (1.001 \pm 0.001)\theta_{GR}$. In the framework of the fourth order gravity theory, the deflection angle of light rays at Sun’s limb depends on both the parameters $\beta$ and $r_c$. We explore this dependence in Fig. 38, by requiring that the expected value for the bending angle is, at least, within $2\sigma$ (grey
region) or within $5\sigma$ (light grey region) the observed value. Inspecting the same figure, it is clear that only $\beta$-values nearby zero (corresponding to a completely arbitrariness of $r_c$) are consistent with the observed deflection angles.

We therefore emphasize that $\beta$-values considered in Capozziello et al. (2006) (i.e. $\beta = 0.25, 0.43, 0.58, 0.75$) are ruled out by light deflection data.
Figure 38: Constraints on the fourth order theory parameters ($\beta$ and $r_c$) arising from the deflection angle of light rays close to the solar limb. The grey and light grey regions embed the part of the parameter space allowed by solar system observations at the $2\sigma$ and $5\sigma$ confidence level, respectively. It is noticing that for scale reason we have not plotted values of $r_c$ up to $10^4$ AU. For these cases, the observations can be restored only for $\beta \to 0$. 
A stronger constraint on the fourth order gravity theory can be obtained from the motion of the solar system planets. Let us consider as a toy model a planet moving on circular orbit (of radius $r$) around the Sun. From Eq. (2), the planet acceleration $a = -\partial \Phi(r)/\partial r$ is given by

$$a = -\frac{Gm}{2r^2} \left[ 1 + \left( \frac{r}{r_c} \right)^\beta - \beta \left( \frac{r}{r_c} \right)^\beta \right].$$

(5)

Accordingly, the planetary circular velocity $v$ can be evaluated and, in turn, the orbital period $P$ is given by

$$P = P_K \sqrt{2} \left[ 1 + \left( \frac{r}{r_c} \right)^\beta - \beta \left( \frac{r}{r_c} \right)^\beta \right]^{-1/2}.$$

(6)

where $P_K = \left[\frac{4\pi^2r^3}{(Gm)}\right]^{1/2}$ is the usual Keplerian period. In order to
compare the orbital period predicted by the fourth order theory with the Solar System observations, let us define the quantity

$$\frac{\Delta P}{P_K} = \left| \frac{P - P_K}{P_K} \right| = |f(\beta, r_c) - 1|$$  \hspace{1cm} (7)

being $f(\beta, r_c)$ the factor appearing on the right hand side of equation (6) and multiplying the usual Keplerian period.

There is a question about a possibility to satisfy the planetary period condition – vanishing the Eq. (7) – with $\beta$ parameter which is significantly different from zero. Vanishing the right hand side of Eq. (7) we obtain the relation

$$\ln r = \ln r_c - \frac{\ln (1 - \beta)}{\beta}$$  \hspace{1cm} (8)

so that Eq. (8) should be satisfied for all the planetary radii. This is obviously impossible since the fourth order theory defines $\beta$ as a parameter,
while the specific system under consideration (the Solar system in our case) allows us to specify the $r_c$ parameter. Hence the right hand side of Eq. (8) is fixed for the Solar system, implying that it is impossible to satisfy Eq. (8) even with two (or more) different planetary radii.
Figure 39: The orbital period in units of the Keplerian one (the function $f(\beta, r_c)$) is given, as a function of $\beta$, in the case of Mercury (dashed line), Venus (dotted line), Earth (solid line) for different $r_c$ (it is clear that $f(\beta, r_c) \to \sqrt{2}$ for $\beta \to 1$). If $r > r_c$ there is $\beta \in (0, 1)$ satisfying Eq. (8), but the $\beta$ values depend on fixed $r_c$ and $r$ and they are different for a fixed $r_c$ and different $r$, so that it is impossible to satisfy Eq. (8) if the number of planets is more than one. Moreover, if we have at least one radius $r \leq r_c$, there is no solution of Eq. (8).
Just for illustration we present the function $f(\beta, r_c)$ as a dependence on $\beta$ parameter for fixed $r_c$ and planetary radii $r$ (see Fig. 39). As one can see from Eq. (8) (and Fig. 39 as well) for each planetary radius $r > r_c$ there is $\beta \in (0, 1)$ satisfying Eq. (8), but the $\beta$ value depends on fixed $r_c$ and $r$, so that they are different for a fixed $r_c$ and different $r$. Moreover, if we have at least one radius $r \leq r_c$, there is no solution of Eq. (8). Both cases imply that the $\beta$ parameter should be around zero.

In Fig. 40 (left panel), the factor $f(\beta, r_c)$ is given as a function of $\beta$ for the two limiting values of $r_c$, 1 AU (dashed line) and $\simeq 10^4$ AU (solid line), considered by Capozziello et al. (2006). As one can note, only for $\beta$ approaching zero it is expected to recover the value of the Keplerian period. In the above mentioned figure, the calculation has been performed for the Earth orbit (i.e. $r = 1$ AU).

Current observations allow also to evaluate the distances between the Sun and the planets of the Solar System with a great accuracy. In
particular, differences in the heliocentric distances do not exceed 10 km for Jupiter and amount to 180, 410, 1200 and 14000 km for Saturn, Uranus, Neptune and Pluto, respectively. Errors in the semi-major axes of the inner planets are even smaller so that the relative error in the orbital period determination is extremely low. As an example, the orbital period of Earth is $T = 365.256363051$ days with an error of $\Delta T = 5.0 \times 10^{-10}$ days, corresponding to a relative error of $\Delta T/T$ less than $10^{-12}$. These values can be used in order to constrain the possible values of both the parameters $\beta$ and $r_c$ introduced by the fourth order gravity theory. This can be done by requiring that $\Delta P/P_K \lesssim \Delta T/T$ so that, in the case of Earth, $|f(\beta, r_c) - 1| \lesssim 10^{-12}$ which can be solved with respect to $\beta$ once the $r_c$ parameter has been fixed to some value. For $r_c = 1$ AU and $r_c = 10^4$ AU (i.e. the two limiting cases considered by Capozziello et al. (2006) we find the allowed upper limits on the $\beta$ parameter to be $4.0 \times 10^{-12}$ and $3.9 \times 10^{-13}$, respectively (since $\Delta P/P_K = \Delta \beta[-1 + \ln (r/r_c)]/4$). These results can also be inferred from the middle and right panels of Fig. 40.
Figure 40: The factor $f(\beta, r_c)$ is given as a function of $\beta$ (left panel) for the two limiting values of $r_c$, 1 AU (dashed line) and $\simeq 10^4$ AU (solid line), respectively. As one can note, only for $\beta$ approaching 0 it is expected to recover the value of the Keplerian period. Here, the calculation have been performed at Earth (i.e. $r = 1$ AU). In the middle and right panel, the quantity $f(\beta, r_c) - 1$ is given as a function of $\beta$ for $r_c = 10^4$ AU and $r_c = \simeq 10^4$ AU. Note that only for values of $\beta$ close to 0 the Solar System observation can be restored (see text for more details).
A more precise analysis which takes into account the planetary semi-major axes and eccentricities leads to variations of at most a few percent with respect to the results in Fig. 40, since the planetary orbits are nearly circular. Therefore, in spite of the fact that orbital periods of planets are not generally used to test alternative theories of gravity (since it is taken for granted that the weak field approximation of these theories gives the Newtonian limit), we found that these data are important to constrain parameters of the fourth order gravity theory.

GR and Newtonian theory (as its weak field limit) were verified by a very precise way at different scales. There are observational data which constrain parameters of alternative theories as well. As a result, the parameter \( \beta \) of fourth order gravity should be very close to zero (it means that the gravitational theory should be very close to GR). In particular, the \( \beta \) parameter values considered for microlensing Capozziello et al. (2006) for rotation curves Capozziello et al. (2006) and cosmological SN type Ia
Borowiec et al. (2006) are ruled out by solar system data.

No doubt that one could also derive further constraints on the fourth order gravity theory by analyzing other physical phenomena such as Shapiro time delay, frequency shift of radio photons, laser ranging for distant objects in the solar system, deviations of trajectories of celestial bodies from ellipses, parabolas and hyperbolas and so on. But our aim was only to show that only $\beta \simeq 0$ values are not in contradiction with solar system data in spite of the fact that there are a lot of speculations to fit observational data with $\beta$ values significantly different from zero.
Conclusions

• Relikt-1, COBE, Anisotropy and Black Body Spectrum. Nobel prize winner (1978) P.L. Kapitza wrote in 1946: "... Our main national defect is an underestimation of our powers and overestimation of foreign ones. So, an extra modesty is much more defective than an extra self-confidence... Very often a cause of unused innovations is that usually we underestimate our own discoveries and overestimate foreign ones..."

• Solar system constraints on $R^n$ theories
Thank you very much for your kind attention